15.3.28, 37

15.2.34, 41

15.2.34

$$f(x,y) = \frac{-x}{\sqrt{x^2+y^2}}$$

Along $y = x$

$$f(x,x) = \frac{-x}{\sqrt{2x^2}} = \frac{-x}{\sqrt{2}\,|x|}$$

As $(x,x) \to (0,0)$ for $x > 0$

$$f(x,x) \to \frac{-1}{\sqrt{2}}$$

for $x < 0$

$$f(x,x) \to \frac{1}{\sqrt{2}}$$

$\Rightarrow$ Limit DNE

15.2.41

$$f(x,y) = \begin{cases} \dfrac{xy}{8x^2+2y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

Definitely continuous for $\mathbb{R}^2 - \{(0,0)\}$

$$\lim_{(x,y)\to(0,0)} \frac{xy}{8x^2+2y^2}$$

Along $y = mx$

$$\frac{x(mx)}{8x^2+2m^2x^2} = \frac{mx^2}{2x^2(4+m^2)}$$

$$= \frac{m}{2(4+m^2)} \longrightarrow \frac{m}{2(4+m^2)}$$

$(x,y)\to(0,0)$

Depends on path $\Rightarrow$ limit DNE

Ex $$f(x,y) = \begin{cases} \dfrac{x^3-y^3}{x^2-xy+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

$$\lim \frac{x^3-y^3}{x^2-xy+y^2} = \lim \frac{(x-y)(x^2-xy+y^2)}{(x^2-xy+y^2)}$$

15.3.28

$$g(x,y) = \cos^7(x^7y^5)$$

$$\frac{\partial g}{\partial x} = 7\cos^6(x^7y^5)\cdot y^5 \cdot 7x^6 \cdot (-\sin(x^7y^5))$$

$$\frac{\partial g}{\partial y} = -7\cos^6(x^7y^5)\sin(x^7y^5)\cdot x^7 \cdot 5y^4$$

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$$G' = g$$

$$f(x,y) = \int_x^y g(t)\,dt$$

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\left(G(y) - G(x)\right)$$

$$= 0 - G'(x)$$

$$= -g(x)$$