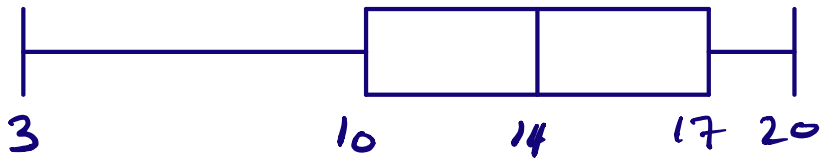
Quiz 4 results



15.7 15.6.63

#3

15.6.63

$$V = \frac{R^2}{L}$$

R: 3 → 3.04

L: 52 → 52.6

$$V_R = \frac{2R}{L}, \quad V_L = -\frac{R^2}{L^2}$$

$$dV = V_R\, dR + V_L\, dL$$

$$dV = \left(\frac{6}{52}\right)(.04) + \left(\frac{-9}{52^2}\right)(.6)$$

15.7.29

$$f(x,y) = 7 + 2x^4 + 5y^4$$

$f_x = 8x^3, \quad f_y = 20y^3$

$f_x = 0 \Rightarrow x = 0$

$f_y = 0 \Rightarrow y = 0$

CP: $(0,0)$ $\quad D = f_{xx} f_{yy} - (f_{xy})^2$

$f_{xx} = 24x^2, \quad f_{yy} = 60y^2, \quad f_{xy} = 0$

$D = (24)(60)x^2y^2$

$D(0,0) = 0$

15.5.35

$$F(x,y) = e^{-\frac{x^2}{5} - \frac{y^2}{5}}, \quad (2,-2)$$

$$\nabla F = \left\langle -\frac{2x}{5} e^{-\frac{x^2}{5} - \frac{y^2}{5}}, \; -\frac{2y}{5} e^{-\frac{x^2}{5} - y^2/5} \right\rangle$$

$$\nabla F(2,-2) = \left\langle -\frac{4}{5} e^{-8/5}, \; \frac{4}{5} e^{-8/5} \right\rangle$$

= Steepest ascent

Steepest descent: $-\nabla F(2,-2)$

The homework problem asks for a unit vector in each of these directions.

$$|\nabla F(2,-2)| = \frac{4}{5}e^{-8/5}|\langle -1, 1\rangle|$$

$$= \frac{4}{5}e^{-8/5} \cdot \sqrt{2}$$

So a unit vector in the direction of $\nabla F(2,-2)$ is

$$\left\langle \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$$

Similarly, a unit vector in the direction of steepest descent is $\langle \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}} \rangle$.

To find a vector pointing in direction of no change, we want $\nabla F \cdot \vec{v} = 0$, or

$$\langle \tfrac{-1}{\sqrt{2}}, \tfrac{1}{\sqrt{2}} \rangle \cdot \langle a, b \rangle = 0$$

$$\Rightarrow a = b.$$

So $\vec{v} = \langle 1, 1 \rangle$ satisfies this.