15.8.26

$$f(x,y,z) = (xyz)^{1/2}$$

$$g = x+y+z = 2, \qquad x \geq 0,\ y \geq 0,\ z \geq 0$$

$$\nabla f = \begin{pmatrix} \frac{1}{2} yz \,(xyz)^{-\frac{1}{2}} \\ \frac{1}{2} xz \,(xyz)^{-\frac{1}{2}} \\ \frac{1}{2} xy \,(xyz)^{-\frac{1}{2}} \end{pmatrix} = \lambda \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \lambda \nabla g$$

$$\frac{1}{2} yz \,(xyz)^{-\frac{1}{2}} = \frac{1}{2} xz \,(xyz)^{\frac{1}{2}} = \frac{1}{2} xy \,(xyz)^{\frac{1}{2}}$$

$$\boxed{yz = xz = xy}$$

First assume $x > 0,\ y > 0,\ z > 0$

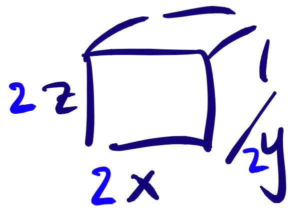
then $z = y = x$ $\qquad (x+y+z=2)$

If $x = 0$, then $\begin{cases} y = 0, & z = 2 \\ y = 2, & z = 0 \end{cases}$

$$x+y+z = x+x+x$$

$$3x = 2 \rightarrow x = \frac{2}{3}$$

* 13.8.27



$x^2+y^2+z^2=1$

$$f(x,y,z)=8xyz$$

$$g(x,y,z)=x^2+y^2+z^2=1$$

$$\nabla f=\begin{pmatrix} yz \\ xz \\ xy \end{pmatrix}=\lambda\begin{pmatrix} 2x \\ 2y \\ 2z \end{pmatrix}=\lambda\nabla g$$

$$xyz=\lambda\cdot 2x^2$$
$$yxz=\lambda\cdot 2y^2$$
$$zxy=\lambda\cdot 2z^2$$

$$x^2=y^2=z^2 \xrightarrow{g} x^2+x^2+x^2=1$$

$$\boxed{x=\frac{1}{\sqrt{3}}}$$

Positive since x is a length