16.2.63

16.2.49

16.3.51

Avg Quiz 5

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16.2.49

$$\iint_R 8xy \, dA$$

R: bounded

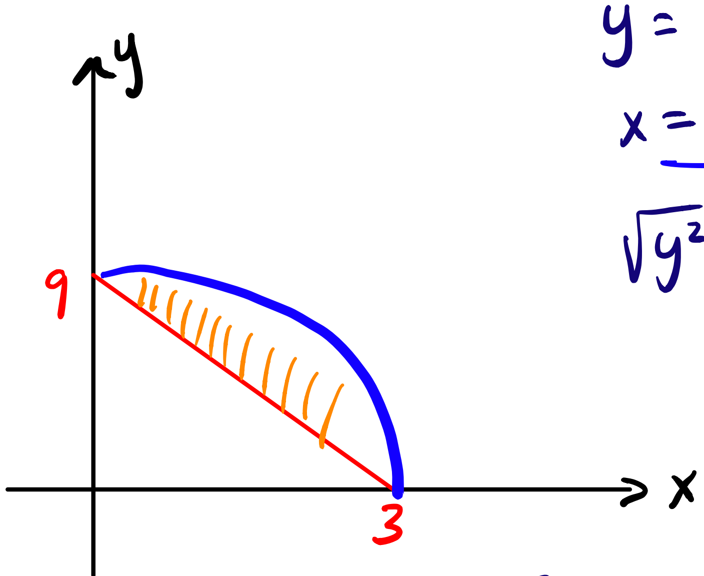
$y = 9 - 3x \rightarrow x = 3 - \frac{1}{3}y$

$y = 0$

1st Quad

$x = 9 - y^2/9$

$\sqrt{y^2} = \sqrt{81 - 9x}$

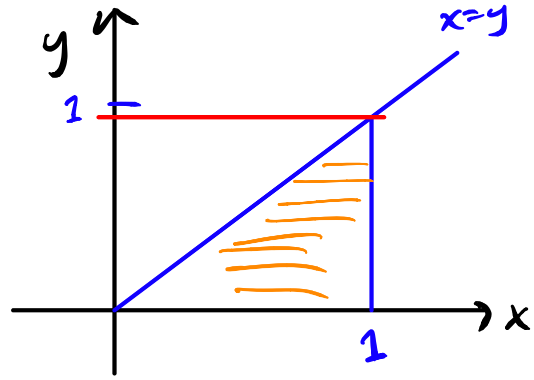


$$\int_0^3 \int_{9-3x}^{\sqrt{81-9x}} 8xy \, dy \, dx$$

$$\int_0^9 \int_{3-\frac{1}{3}y}^{9-y^2/9} 8xy \, dx \, dy$$

16.2.63

$$\int_0^1 \int_y^1 3e^{x^2}\,dx\,dy$$

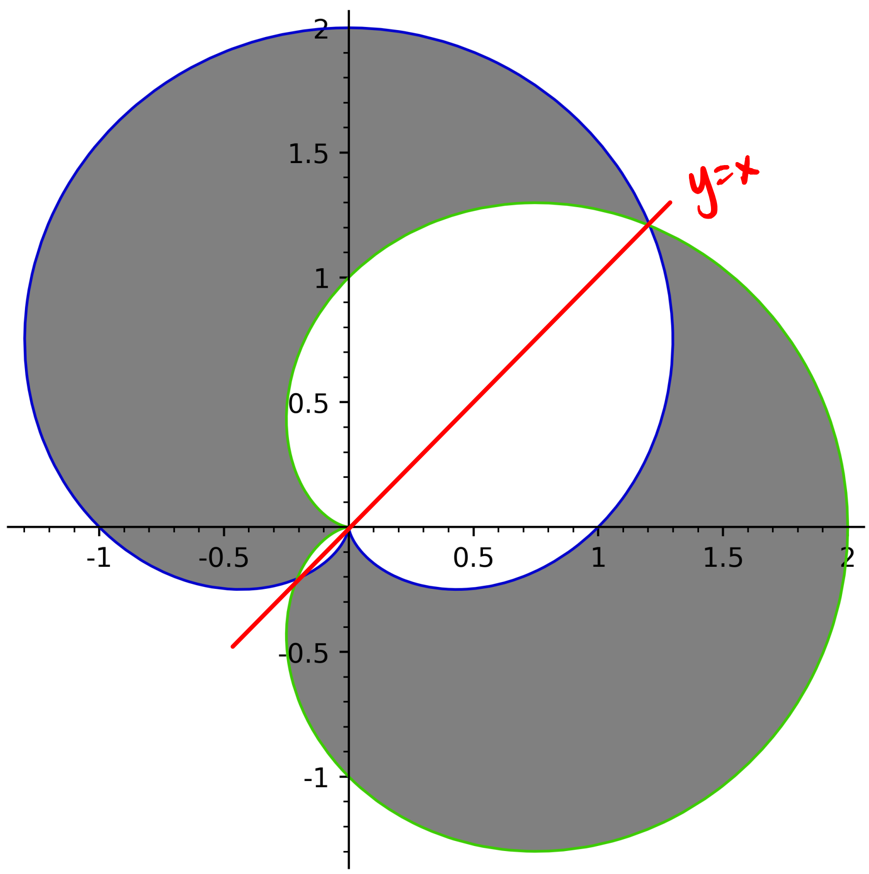


$$\int_0^1 \int_0^x 3e^{x^2}\,dy\,dx$$

$$= \int_0^1 3x e^{x^2}\,dx$$

16.3.51

Area inside $r = 1+\sin\theta$ and $r = 1+\cos\theta$



We want to find the area of the white region. Note that this is symmetric about the line $y=x$. So we can find the area of the region between $r = 1+\cos\theta$ and $y=x$, then multiply by 2.

Finding points of intersection

$$1+\cos\theta = 1+\sin\theta \Rightarrow \cos\theta = \sin\theta$$
$$\Rightarrow \theta = \frac{\pi}{4}, \frac{5\pi}{4}$$

So $\frac{\pi}{4} \le \theta \le \frac{5\pi}{4}$,

and $0 \le r \le 1+\cos\theta$.

$$2\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \int_0^{1+\cos\theta} r \, dr \, d\theta$$

$$= \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (1+\cos\theta)^2 \, d\theta$$

$$= \int_{\pi/4}^{5\pi/4} (1 + 2\cos\theta + \cos^2\theta) \, d\theta$$

$$= \int_{\pi/4}^{5\pi/4} \left[1 + 2\cos\theta + \frac{1}{2}(1+\cos 2\theta)\right] d\theta$$

$$= \boxed{\frac{3\pi}{2} - 2\sqrt{2}}$$