16.3.32

16.3.14

16.3.28

16.1.29

Quiz 5

Aug 18

16.1.29

$$\iint_R \sqrt{\frac{x}{y}}\, dA \qquad R = \{(x,y) \mid 0 \le x \le 4,\ 1 \le y \le 16\}$$

$$\int_1^{16}\int_0^4 \sqrt{\frac{x}{y}}\, dx\, dy = \int_1^{16} \left(y \frac{2}{3} \left(\frac{x}{y}\right)^{3/2} \Bigg|_{x=0}^{x=4} \right) dy$$

$u = \frac{x}{y}$

$du = \frac{1}{y}\, dx$

$$= \frac{2}{3} \int_1^{16} 8\, y \left(\frac{1}{y}\right)^{3/2} dy$$

$$= \frac{16}{3} \int_1^{16} y^{-\frac{1}{2}}\, dy = \#$$

16.3.14 Volume of surface bounded by

$$z = \frac{68}{1+x^2+y^2} - 4 \quad \text{and} \quad xy\text{-plane}$$

$$\iint_R \left(\frac{68}{1+x^2+y^2} - 4 \right) dA$$

Set $z = 0$:

$$4 = \frac{68}{1+x^2+y^2}$$

$$1 + x^2 + y^2 = 17 \rightarrow x^2 + y^2 = 16$$

$$R = \{ (x,y) \mid x^2 + y^2 = 16 \}$$

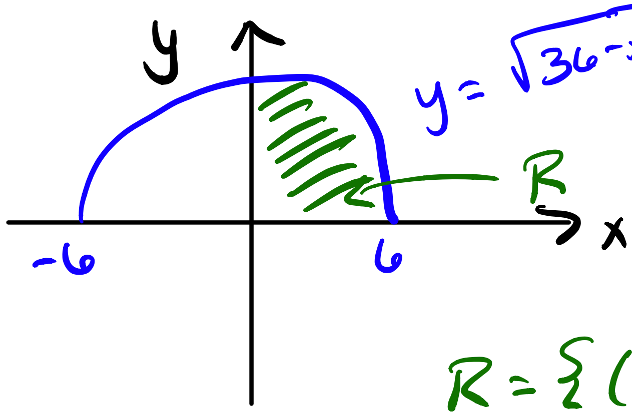
$$= \{ (r,\theta) \mid 0 \le r \le 4, \ 0 \le \theta \le 2\pi \}$$

$$\int_0^{2\pi}\int_0^4 \left(\frac{68}{1+r^2} - 4\right) r\,dr\,d\theta$$

$$= 2\pi\left[\,68\underbrace{\int_0^4 \frac{r}{1+r^2}\,dr}_{\substack{u = 1+r^2 \\ du = 2r\,dr}} - \underbrace{\int_0^4 4r\,dr}_{\text{power rule}}\,\right]$$

16.3.28

$$\int_0^6 \int_0^{\sqrt{36-x^2}} \sqrt{x^2+y^2}\;dy\,dx$$



$y = \sqrt{36-x^2}$ (half-circle)

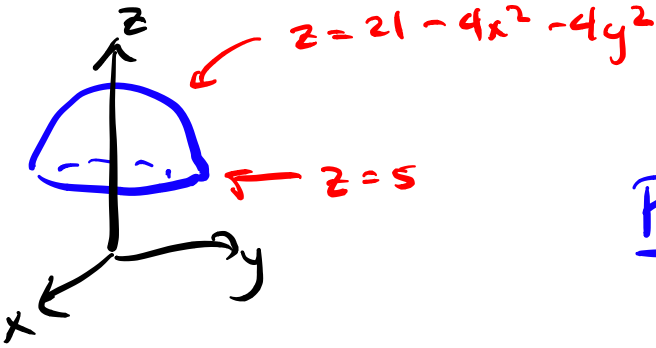
$R = \{(r,\theta) \mid 0 \le r \le 6,\ 0 \le \theta \le \frac{\pi}{2}\}$

$$\int_0^{\frac{\pi}{2}}\int_0^6 r^2\,dr\,d\theta = \frac{\pi}{2}\int_0^6 r^2\,dr$$

$$= 36\pi$$

16.3.32

Volume of solid bdd by $z = 21 - 4x^2 - 4y^2$ and $z = 5$



Polar

$z = 21 - 4r^2$

$z = 5$

$$\iint_R \left[(21 - 4r^2) - 5 \right] dA$$

$$= 4 \iint_R 4 - r^2 \, dA$$

$R = \{ (r, \theta) \mid 0 \le r \le 2, \ 0 \le \theta \le 2\pi \}$

Set $5 = 21 - r^2 \rightarrow 4r^2 = 16 \rightarrow r^2 = 4$

$$4 \int_0^{2\pi} \int_0^2 (4 - r^2) \, r \, dr \, d\theta$$