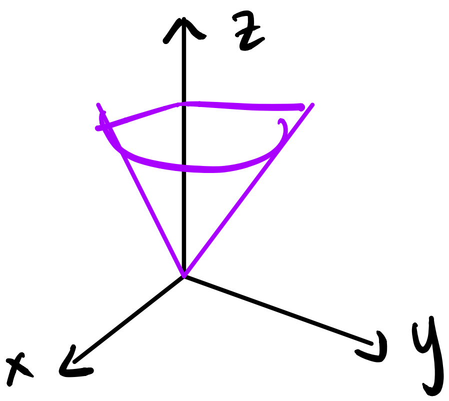
Less 22 #6

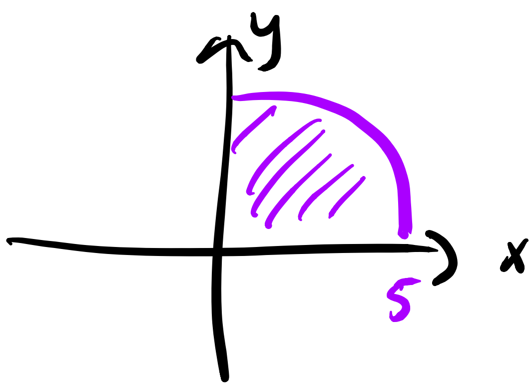
16.5.49 , * 14.7.63, 61, 53

16.5.21

$$\int_0^5 \int_0^{\sqrt{25-x^2}} \int_0^{\sqrt{x^2+y^2}} (x^2+y^2)^{-1/2}\, dz\,dy\,dx$$

$z = \sqrt{x^2+y^2}$

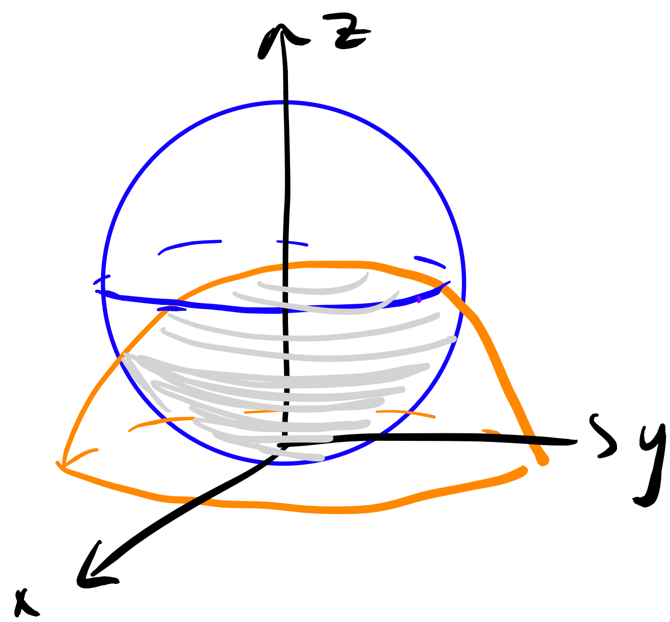


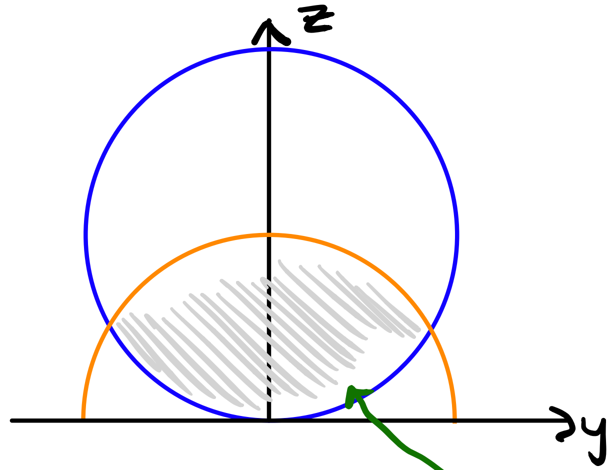


$$\int_0^{\frac{\pi}{2}} \int_0^5 \int_0^r r^{-1} \cdot r \, dz\,dr\,d\theta$$

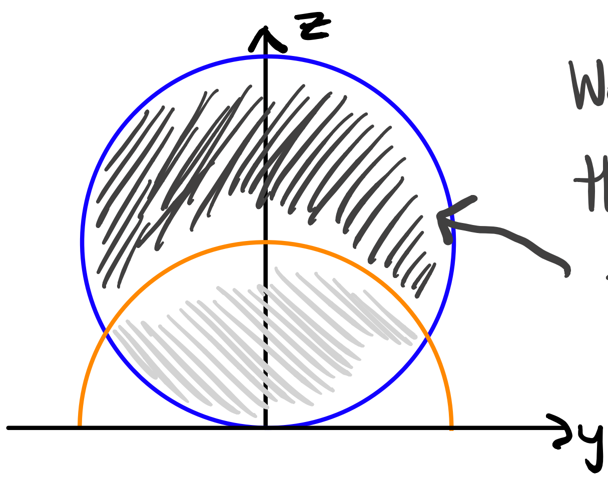
16.5.49

Region bdd by $\rho = 4\cos\varphi$, $\rho = 2$, $z \geq 0$

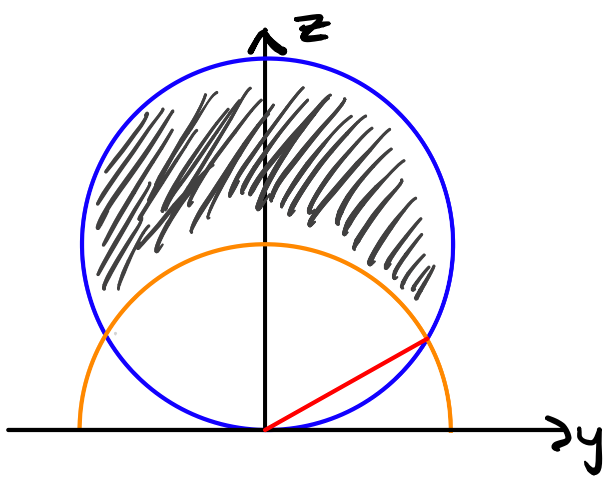




We want this region revolved about the z-axis



We can find the volume of the blue sphere, then take away the volume corresponding to this region



Intersection occurs when

$$2 = \rho = 4\cos\varphi \rightarrow \varphi = \frac{\pi}{3}$$

So $0 \le \varphi \le \frac{\pi}{3}$, and ρ ranges from the orange sphere to the blue sphere, that is,

$$2 \le \rho \le 4\cos\varphi.$$

And $0 \le \theta \le 2\pi$. This gives

$$\int_0^{2\pi} \int_0^{\frac{\pi}{3}} \int_2^{4\cos\varphi} \rho^2 \sin\varphi \, d\rho \, d\varphi \, d\theta$$

$$= \frac{22\pi}{3}.$$

Now the volume of the whole blue sphere is $V = \frac{4}{3}\pi(2)^3 = \frac{32\pi}{3}$

so the volume of the gray region is

$$\frac{32\pi}{3} - \frac{22\pi}{3} = \boxed{\frac{10\pi}{3}}$$

* 14.7.61 Vol of sphere $\rho = 9$

Spherical

$$\int_0^{2\pi} \int_0^{\pi} \int_0^{9} \rho^2 \sin\varphi \, d\rho \, d\varphi \, d\theta$$

or $$8 \int_0^{\frac{\pi}{2}} \int_0^{\pi/2} \int_0^{9} \rho^2 \sin\varphi \, d\rho \, d\varphi \, d\theta$$

cylindrical

$$8 \underbrace{\int_0^{\frac{\pi}{2}} \int_0^{9}}_{\text{project to } xy\text{-plane}} \int_0^{\sqrt{81-r^2}} r \, dz \, dr \, d\theta$$

Rectangular

$$8 \int_0^{9} \int_0^{\sqrt{81-x^2}} \int_0^{\sqrt{81-x^2-y^2}} dz \, dy \, dx$$