Instructions. Show all work, with clear logical steps. No work or hard-to-follow work will lose points.

1.) (5 points) Compute the orthogonal projection of $\mathbf{v} = \langle -1, 3, 2\rangle$ onto $\mathbf{w} = \langle 4, 2, 2\rangle$.

Solution.

$$\begin{aligned}\text{proj}_{\mathbf{w}}\mathbf{v} &= \frac{\mathbf{v}\cdot\mathbf{w}}{\mathbf{w}\cdot\mathbf{w}}\mathbf{w} = \frac{-4+6+4}{16+4+4}\langle 4,2,2\rangle \\ &= \frac{1}{4}\langle 4,2,2\rangle.\end{aligned}$$

□

2.) (5 points) Find a vector that is orthogonal to both $\mathbf{v} = \langle -1, 3, 2\rangle$ and $\mathbf{w} = \langle 4, 2, 2\rangle$.

Solution. This is given by $\mathbf{v}\times\mathbf{w}$.

$$\begin{aligned}\mathbf{v}\times\mathbf{w} &= \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ -1 & 3 & 2\\ 4 & 2 & 2\end{vmatrix} = \begin{vmatrix}3 & 2\\ 2 & 2\end{vmatrix}\mathbf{i} - \begin{vmatrix}-1 & 2\\ 4 & 2\end{vmatrix}\mathbf{j} + \begin{vmatrix}-1 & 3\\ 4 & 2\end{vmatrix}\mathbf{k} \\ &= (6-4)\mathbf{i} - (-2-8)\mathbf{j} + (-2-12)\mathbf{k} \\ &= 2\mathbf{i} + 10\mathbf{j} - 14\mathbf{k} \\ &= \langle 2, 10, -14\rangle\end{aligned}$$

□

3.) (5 points) Find the equation of the plane that is parallel to the vectors $\mathbf{v} = \langle -1, 3, 2\rangle$ and $\mathbf{w} = \langle 4, 2, 2\rangle$ passing through the point $(2, 0, -2)$.

Solution. In the previous problem we found that $\mathbf{n} = \langle 2, 10, -14\rangle$ is a normal vector for this plane. So the equation of the plane is given by

$$\begin{aligned}0 &= \mathbf{n}\cdot\langle x-2, y, z+2\rangle \\ &= \langle 2, 10, -14\rangle\cdot\langle x-2, y, z+2\rangle \\ &= 2(x-2) + 10y - 14(z+2).\end{aligned}$$

Or in general form, $2x + 10y - 14z = 32$. □

4.) (5 points) Identify the surface defined by

$$x^2 + y^2 + 6z^2 + 6x = -8.$$

Solution. We start by completing the square.

$$\begin{aligned}x^2 + y^2 + 6z^2 + 6x &= -8 \\ x^2 + 6x + 9 + y^2 + 6z^2 &= -8 + 9 \\ (x+3)^2 + y^2 + 6z^2 &= 1 \\ (x+3)^2 + y^2 + \frac{z^2}{(1/\sqrt{6})^2} &= 1\end{aligned}$$

So this is an ellipsoid with center $(-3, 0, 0)$ that has length 2 in the x- and y-directions and length $2/\sqrt{6}$ in the z-direction. □