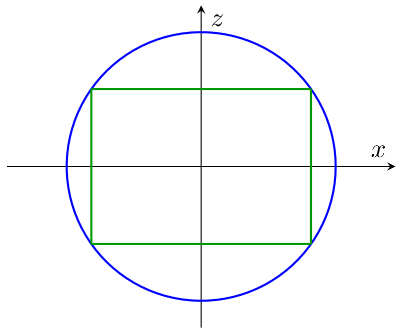
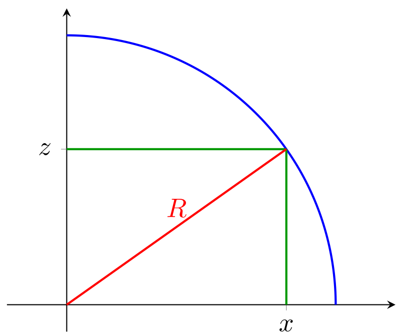
Instructions. Show all work, with clear logical steps. No work or hard-to-follow work will lose points.

1.) (6 points) Find the dimensions of the right circular cylinder of maximum volume that can be inscribed in a sphere of radius 16.

Solution. It helps to draw a picture. Cutting the sphere in half, a cross section looks like this.





The volume of the inscribed cylinder is $V = \pi r^2 h$. Since the cylinder is centered inside the sphere, we have $x = r$ and $2z = h$. So we have a volume function

$$V(x, z) = 2\pi x^2 z.$$

Looking at the projection onto the xz-axis, we must have $g(x, z) = x^2 + z^2 = 16^2$. Now

$$V_x = 4\pi xz = 2x\lambda = \lambda g_x \qquad (1)$$
$$V_z = 2\pi x^2 = 2z\lambda = \lambda g_z. \qquad (2)$$

Solving (1) for λ, and substituting into (2), we see that $x^2 = 2z^2$. Using this in the constraint $g(x, z)$ gives

$$\begin{aligned} 16^2 &= x^2 + z^2 \\ &= x^2 + 2z^2 \\ &= 3z^2, \end{aligned}$$

which implies that $z = \frac{16}{\sqrt{3}}$, and so the height of the cylinder is $\frac{32}{\sqrt{3}}$. Since $x^2 = 2z^2$, we find $x = \frac{16\sqrt{2}}{\sqrt{3}}$. $\square$

2.) (6 points) Evaluate the integral

$$\iint_R e^{x+y}\, dA,$$

where $R = \{(x, y) \mid 0 \le x \le \ln 3,\ 1 \le y \le \ln 5\}$.

Solution.

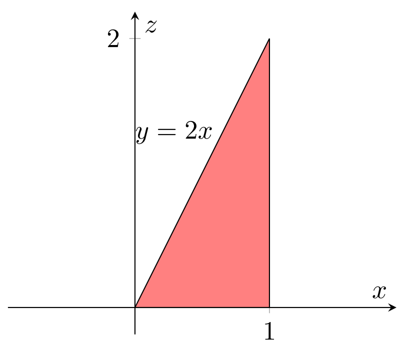
$$\begin{aligned}\iint e^{x+y}\, dA &= \int_1^{\ln 5}\int_0^{\ln 3} e^{x+y}\, dxdy \\ &= \int \left(e^{x+y}\Big|_{x=0}^{x=\ln 3} \right) dy \\ &= \int_1^{\ln 5} \left(e^{y+\ln 3} - e^y\right) dy \\ &= \left(e^{y+\ln 3} - e^y\right)\Big|_1^{\ln 5} \\ &= 10 - 2e.\end{aligned}$$

□

3.) (6 points) Evaluate the integral

$$\int_0^2 \int_{y/2}^1 e^{x^2}\, dxdy$$

Solution. Drawing the domain of integration:



$$\begin{aligned}\int_0^2 \int_{y/2}^1 e^{x^2}\, dxdy &= \int_0^1 \int_0^{2x} e^{x^2}\, dydx \\ &= \int_0^1 2xe^{x^2}\, dx \\ &= e^{x^2}\Big|_0^1 \\ &= e - 1.\end{aligned}$$

□

4.) (2 points) What's your favorite Dr. Seuss book?

Solution. Fox in Socks.

□