Instructions. Show all work, with clear logical steps. No work or hard-to-follow work will lose points.

1.) (9 points) Find the volume of the solid bounded by

$$z = 22 - \sqrt{x^2 + y^2}$$

and the xy-plane.

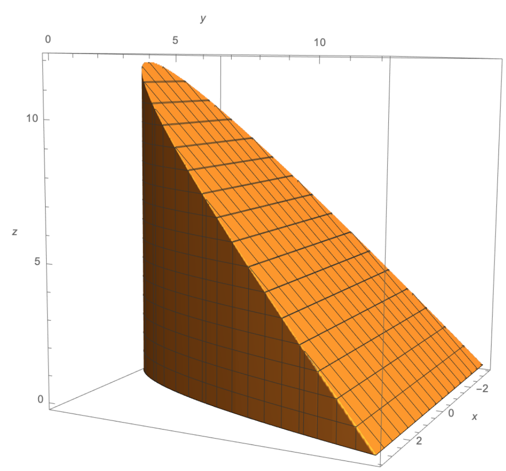
Solution. (The boundary of) our domain of integration in the xy-plane is given by the equation $0 = 22 - \sqrt{x^2 + y^2}$. So $D = \{(x, y) \mid x^2 + y^2 \leq 22^2\} = \{(r, \theta) \mid 0 \leq r \leq 22,\ 0 \leq \theta \leq 2\pi\}$. So the volume of the solid is given by

$$\begin{aligned} V &= \iint_D \left(22 - \sqrt{x^2 + y^2}\right) dA \\ &= \int_0^{2\pi} \int_0^{22} (22 - r) r \, dr d\theta \\ &= 2\pi \int_0^{22} (22r - r^2) \, dr \\ &= 2\pi \left(11r^2 - \frac{1}{3} r^3\right) \Big|_0^{22} \\ &= \frac{10648\pi}{3} \end{aligned}$$

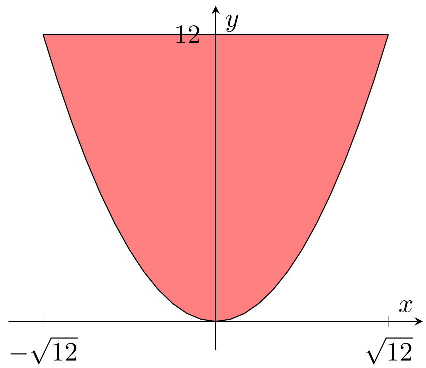
□

2.) (9 points) Use a triple integral to find the volume of the wedge bounded by the parabolic cylinder $y = x^2$ and the planes $z = 12 - y$ and $z = 0$.

Solution. It helps to draw a picture of the given region.



And looking at the projection in the xy-plane, we have



So then $0 \le x \le \sqrt{12}$ and $0 \le y \le x^2$, while $0 \le z \le 12 - y$. This gives the triple integral

$$\begin{aligned}
\int_{-\sqrt{12}}^{\sqrt{12}} \int_{x^2}^{12} \int_0^{12-y} dzdydx &= 2\int_0^{\sqrt{12}} \int_{x^2}^{12} (12-y)\, dydx \\
&= 2\int_0^{\sqrt{12}} \left(12y - y^2/2\right)\Big|_{y=x^2}^{y=12} dx \\
&= 2\int_0^{\sqrt{12}} \left(72 - 12x^2 + x^4/2\right) dx \\
&= 144x - 8x^3 + \frac{x^5}{5}\Big|_0^{\sqrt{12}} \\
&= 144\sqrt{12} - 96\sqrt{12} + \frac{144}{5}\sqrt{12} \\
&= \frac{384}{5}\sqrt{12} \\
&= \frac{768}{5}\sqrt{3}.
\end{aligned}$$

□

3.) (2 points) Do you have any fun plans for spring break?

Solution. I did but they were canceled. □