Cut here and keep for reference

REFERENCE page 1

ALGEBRA

Arithmetic Operations

$a(b + c) = ab + ac$

$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$

$\frac{a + c}{b} = \frac{a}{b} + \frac{c}{b}$

$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$

Exponents and Radicals

$x^m x^n = x^{m+n}$

$\frac{x^m}{x^n} = x^{m-n}$

$(x^m)^n = x^{mn}$

$x^{-n} = \frac{1}{x^n}$

$(xy)^n = x^n y^n$

$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$

$x^{1/n} = \sqrt[n]{x}$

$x^{m/n} = \sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m$

$\sqrt[n]{xy} = \sqrt[n]{x}\,\sqrt[n]{y}$

$\sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}$

Factoring Special Polynomials

$x^2 - y^2 = (x + y)(x - y)$

$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

Binomial Theorem

$(x + y)^2 = x^2 + 2xy + y^2$ $\qquad (x - y)^2 = x^2 - 2xy + y^2$

$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$

$$(x + y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{2}x^{n-2}y^2 + \cdots + \binom{n}{k}x^{n-k}y^k + \cdots + nxy^{n-1} + y^n$$

where $\binom{n}{k} = \frac{n(n-1)\cdots(n-k+1)}{1 \cdot 2 \cdot 3 \cdot \cdots \cdot k}$

Quadratic Formula

If $ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Inequalities and Absolute Value

If $a < b$ and $b < c$, then $a < c$.

If $a < b$, then $a + c < b + c$.

If $a < b$ and $c > 0$, then $ca < cb$.

If $a < b$ and $c < 0$, then $ca > cb$.

If $a > 0$, then

$|x| = a$ means $x = a$ or $x = -a$

$|x| < a$ means $-a < x < a$

$|x| > a$ means $x > a$ or $x < -a$

GEOMETRY

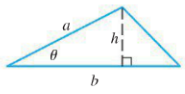
Geometric Formulas

Formulas for area A, circumference C, and volume V:

Triangle

$A = \frac{1}{2}bh$

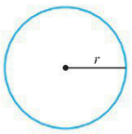
$= \frac{1}{2}ab \sin\theta$



Circle

$A = \pi r^2$

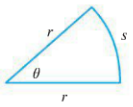
$C = 2\pi r$



Sector of Circle

$A = \frac{1}{2}r^2\theta$

$s = r\theta$ (θ in radians)



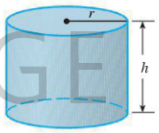
Sphere

$V = \frac{4}{3}\pi r^3$

$A = 4\pi r^2$

Cylinder

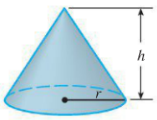
$V = \pi r^2 h$



Cone

$V = \frac{1}{3}\pi r^2 h$

$A = \pi r\sqrt{r^2 + h^2}$



Distance and Midpoint Formulas

Distance between $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint of $\overline{P_1P_2}$: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Lines

Slope of line through $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Point-slope equation of line through $P_1(x_1, y_1)$ with slope m:

$$y - y_1 = m(x - x_1)$$

Slope-intercept equation of line with slope m and y-intercept b:

$$y = mx + b$$

Circles

Equation of the circle with center (h, k) and radius r:

$$(x - h)^2 + (y - k)^2 = r^2$$

TRIGONOMETRY

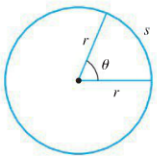
Angle Measurement

π radians $= 180°$

$1° = \frac{\pi}{180}$ rad $\qquad 1 \text{ rad} = \frac{180°}{\pi}$

$s = r\theta$

(θ in radians)

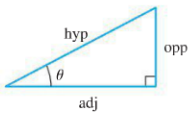


Right Angle Trigonometry

$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}}$

$\cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}}$

$\tan\theta = \frac{\text{opp}}{\text{adj}} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}}$

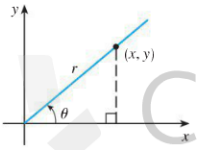


Trigonometric Functions

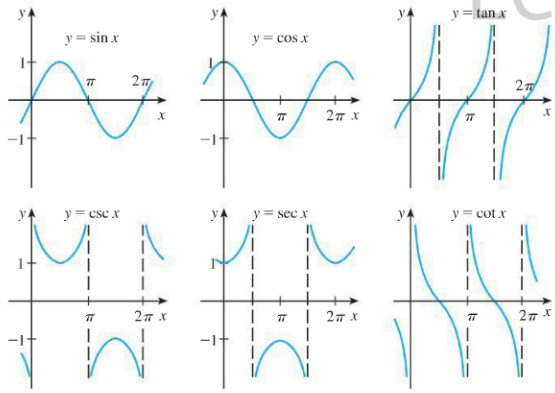
$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{r}{y}$

$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{r}{x}$

$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$



Graphs of Trigonometric Functions



Trigonometric Functions of Important Angles

θ	radians	$\sin\theta$	$\cos\theta$	$\tan\theta$
$0°$	0	0	1	0
$30°$	$\pi/6$	$1/2$	$\sqrt{3}/2$	$\sqrt{3}/3$
$45°$	$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	1
$60°$	$\pi/3$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
$90°$	$\pi/2$	1	0	—

Fundamental Identities

$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$

$\cot\theta = \frac{1}{\tan\theta} \qquad \sin^2\theta + \cos^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta \qquad 1 + \cot^2\theta = \csc^2\theta$

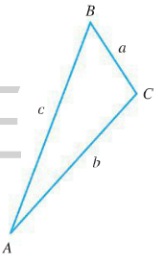
$\sin(-\theta) = -\sin\theta \qquad \cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta \qquad \sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$

$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta \qquad \tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$

The Law of Sines

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$



The Law of Cosines

$a^2 = b^2 + c^2 - 2bc\cos A$

$b^2 = a^2 + c^2 - 2ac\cos B$

$c^2 = a^2 + b^2 - 2ab\cos C$

Addition and Subtraction Formulas

$\sin(x + y) = \sin x\cos y + \cos x\sin y$

$\sin(x - y) = \sin x\cos y - \cos x\sin y$

$\cos(x + y) = \cos x\cos y - \sin x\sin y$

$\cos(x - y) = \cos x\cos y + \sin x\sin y$

$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x\tan y}$

$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x\tan y}$

Double-Angle Formulas

$\sin 2x = 2\sin x\cos x$

$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$

$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$

Half-Angle Formulas

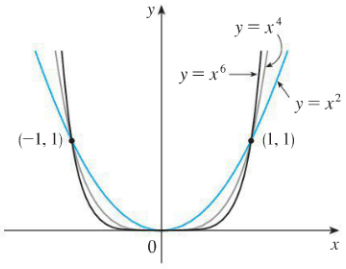
$\sin^2 x = \frac{1 - \cos 2x}{2} \qquad \cos^2 x = \frac{1 + \cos 2x}{2}$

Cut here and keep for reference

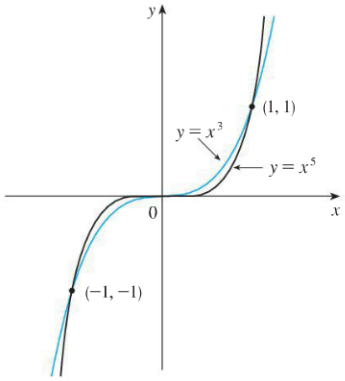
SPECIAL FUNCTIONS

Power Functions $f(x) = x^a$

(i) $f(x) = x^n$, n a positive integer

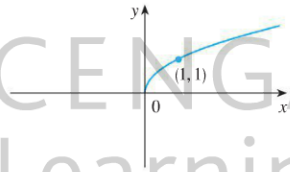


n even

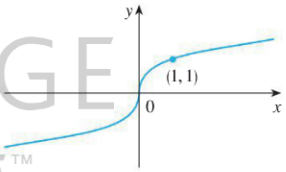


n odd

(ii) $f(x) = x^{1/n} = \sqrt[n]{x}$, n a positive integer

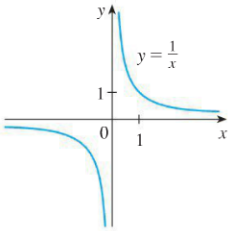


$f(x) = \sqrt{x}$



$f(x) = \sqrt[3]{x}$

(iii) $f(x) = x^{-1} = \dfrac{1}{x}$

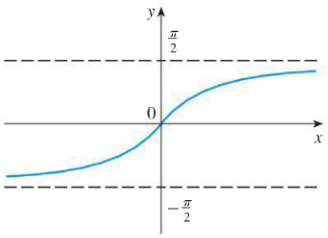


Inverse Trigonometric Functions

$\arcsin x = \sin^{-1} x = y \iff \sin y = x \quad \text{and} \quad -\dfrac{\pi}{2} \leqslant y \leqslant \dfrac{\pi}{2}$

$\arccos x = \cos^{-1} x = y \iff \cos y = x \quad \text{and} \quad 0 \leqslant y \leqslant \pi$

$\arctan x = \tan^{-1} x = y \iff \tan y = x \quad \text{and} \quad -\dfrac{\pi}{2} < y < \dfrac{\pi}{2}$



$y = \tan^{-1} x = \arctan x$

$\lim_{x \to -\infty} \tan^{-1} x = -\dfrac{\pi}{2}$

$\lim_{x \to \infty} \tan^{-1} x = \dfrac{\pi}{2}$

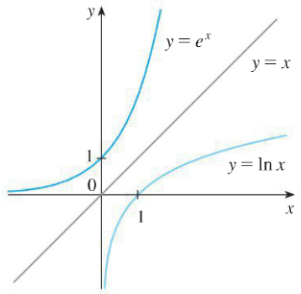
SPECIAL FUNCTIONS

Exponential and Logarithmic Functions

$\log_b x = y \iff b^y = x$

$\ln x = \log_e x$, where $\ln e = 1$

$\ln x = y \iff e^y = x$



Cancellation Equations

$\log_b(b^x) = x \qquad b^{\log_b x} = x$

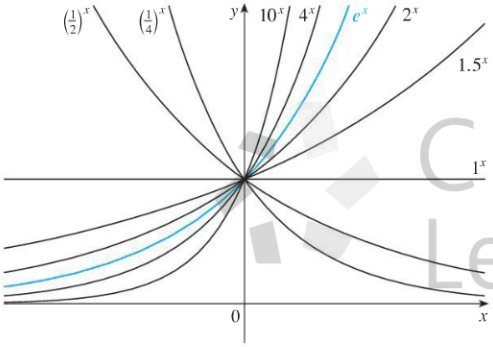
$\ln(e^x) = x \qquad e^{\ln x} = x$

Laws of Logarithms

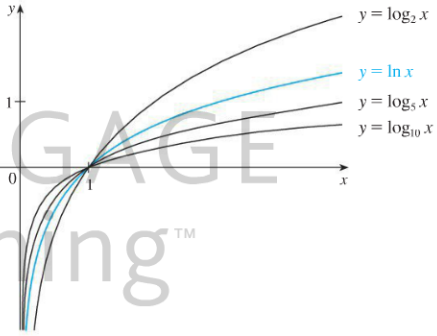
1. $\log_b(xy) = \log_b x + \log_b y$
2. $\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$
3. $\log_b(x^r) = r \log_b x$

$\lim_{x \to -\infty} e^x = 0 \qquad \lim_{x \to \infty} e^x = \infty$

$\lim_{x \to 0^+} \ln x = -\infty \qquad \lim_{x \to \infty} \ln x = \infty$



Exponential functions



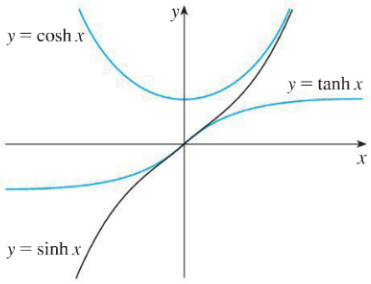
Logarithmic functions

Hyperbolic Functions

$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \operatorname{csch} x = \frac{1}{\sinh x}$

$\cosh x = \frac{e^x + e^{-x}}{2} \qquad \operatorname{sech} x = \frac{1}{\cosh x}$

$\tanh x = \frac{\sinh x}{\cosh x} \qquad \coth x = \frac{\cosh x}{\sinh x}$



Inverse Hyperbolic Functions

$y = \sinh^{-1} x \iff \sinh y = x \qquad \sinh^{-1} x = \ln\left(x + \sqrt{x^2 + 1}\right)$

$y = \cosh^{-1} x \iff \cosh y = x \text{ and } y \geq 0 \qquad \cosh^{-1} x = \ln\left(x + \sqrt{x^2 - 1}\right)$

$y = \tanh^{-1} x \iff \tanh y = x \qquad \tanh^{-1} x = \frac{1}{2} \ln\left(\frac{1 + x}{1 - x}\right)$

Cut here and keep for reference

DIFFERENTIATION RULES

General Formulas

1. $\frac{d}{dx}(c) = 0$
2. $\frac{d}{dx}[cf(x)] = cf'(x)$
3. $\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$
4. $\frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$
5. $\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$ (Product Rule)
6. $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$ (Quotient Rule)
7. $\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$ (Chain Rule)
8. $\frac{d}{dx}(x^n) = nx^{n-1}$ (Power Rule)

Exponential and Logarithmic Functions

9. $\frac{d}{dx}(e^x) = e^x$
10. $\frac{d}{dx}(b^x) = b^x \ln b$
11. $\frac{d}{dx}\ln|x| = \frac{1}{x}$
12. $\frac{d}{dx}(\log_b x) = \frac{1}{x \ln b}$

Trigonometric Functions

13. $\frac{d}{dx}(\sin x) = \cos x$
14. $\frac{d}{dx}(\cos x) = -\sin x$
15. $\frac{d}{dx}(\tan x) = \sec^2 x$
16. $\frac{d}{dx}(\csc x) = -\csc x \cot x$
17. $\frac{d}{dx}(\sec x) = \sec x \tan x$
18. $\frac{d}{dx}(\cot x) = -\csc^2 x$

Inverse Trigonometric Functions

19. $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1 - x^2}}$
20. $\frac{d}{dx}(\cos^{-1}x) = -\frac{1}{\sqrt{1 - x^2}}$
21. $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1 + x^2}$
22. $\frac{d}{dx}(\csc^{-1}x) = -\frac{1}{x\sqrt{x^2 - 1}}$
23. $\frac{d}{dx}(\sec^{-1}x) = \frac{1}{x\sqrt{x^2 - 1}}$
24. $\frac{d}{dx}(\cot^{-1}x) = -\frac{1}{1 + x^2}$

Hyperbolic Functions

25. $\frac{d}{dx}(\sinh x) = \cosh x$
26. $\frac{d}{dx}(\cosh x) = \sinh x$
27. $\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$
28. $\frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth x$
29. $\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$
30. $\frac{d}{dx}(\coth x) = -\operatorname{csch}^2 x$

Inverse Hyperbolic Functions

31. $\frac{d}{dx}(\sinh^{-1}x) = \frac{1}{\sqrt{1 + x^2}}$
32. $\frac{d}{dx}(\cosh^{-1}x) = \frac{1}{\sqrt{x^2 - 1}}$
33. $\frac{d}{dx}(\tanh^{-1}x) = \frac{1}{1 - x^2}$
34. $\frac{d}{dx}(\operatorname{csch}^{-1}x) = -\frac{1}{|x|\sqrt{x^2 + 1}}$
35. $\frac{d}{dx}(\operatorname{sech}^{-1}x) = -\frac{1}{x\sqrt{1 - x^2}}$
36. $\frac{d}{dx}(\coth^{-1}x) = \frac{1}{1 - x^2}$

TABLE OF INTEGRALS

Basic Forms

1. $\int u\,dv = uv - \int v\,du$
2. $\int u^n\,du = \dfrac{u^{n+1}}{n+1} + C, \quad n \neq -1$
3. $\int \dfrac{du}{u} = \ln|u| + C$
4. $\int e^u\,du = e^u + C$
5. $\int b^u\,du = \dfrac{b^u}{\ln b} + C$
6. $\int \sin u\,du = -\cos u + C$
7. $\int \cos u\,du = \sin u + C$
8. $\int \sec^2 u\,du = \tan u + C$
9. $\int \csc^2 u\,du = -\cot u + C$
10. $\int \sec u \tan u\,du = \sec u + C$
11. $\int \csc u \cot u\,du = -\csc u + C$
12. $\int \tan u\,du = \ln|\sec u| + C$
13. $\int \cot u\,du = \ln|\sin u| + C$
14. $\int \sec u\,du = \ln|\sec u + \tan u| + C$
15. $\int \csc u\,du = \ln|\csc u - \cot u| + C$
16. $\int \dfrac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}\dfrac{u}{a} + C, \quad a > 0$
17. $\int \dfrac{du}{a^2 + u^2} = \dfrac{1}{a}\tan^{-1}\dfrac{u}{a} + C$
18. $\int \dfrac{du}{u\sqrt{u^2 - a^2}} = \dfrac{1}{a}\sec^{-1}\dfrac{u}{a} + C$
19. $\int \dfrac{du}{a^2 - u^2} = \dfrac{1}{2a}\ln\left|\dfrac{u + a}{u - a}\right| + C$
20. $\int \dfrac{du}{u^2 - a^2} = \dfrac{1}{2a}\ln\left|\dfrac{u - a}{u + a}\right| + C$