**MA 16200** **Midterm Exam 1** **Summer 2017**

**Name:** _____

**ID number:** _____

Instructions:

1. This is a one-hour exam.
2. There are 10 problems on this exam.
3. No books, notes, or calculators are allowed.
4. Please turn off your cell phone.
5. Circle one and only one choice for each multiple-choice problem. No partial credit will be given for multiple-choice problems.
6. Show all relevant work on non-multiple-choice problems. Partial credit will be given for steps leading to the correct solutions. Write your final answer in the box provided.
7. You may use a writing utensil, your own brain and the paper provided in this exam. Use of any other persons or resources will be considered cheating and will be reported to the Office of the Dean of Students.

I agree to abide by the instructions above:

**Signature:** _____

**Useful trig formulas**:

$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$ $\sin x \cos x = \frac{1}{2}\sin 2x$

**1.** (6 points) Find a vector perpendicular to the plane containing the points $P(4,3,7)$, $Q(-3,0,1)$ and $R(2,-6,5)$.

A. $\langle 3,-25,9\rangle$

B. $\langle 57,-6,-30\rangle$

C. $\langle 6,17,18\rangle$

D. $\langle 14,27,12\rangle$

E. $\langle -48,-2,57\rangle$

**2.** (6 points) Given that $|\mathbf{a}|=4$, $|\mathbf{b}|=7$ and the angle between $\mathbf{a}$ and $\mathbf{b}$ is $\frac{\pi}{6}$, what is $\mathbf{a}\cdot\mathbf{b}$?

A. 14

B. $14\sqrt{2}$

C. $14\sqrt{3}$

D. $14\sqrt{6}$

E. 28

**3.** (6 points) The partial fraction decomposition of

$$\frac{3x^2 - x + 4}{x^4 - x^3 + 2x^2 - 2x}$$

has the form

$$\frac{A}{x} + \frac{B}{x-1} + \frac{Cx + D}{x^2 + 2}.$$

What is $3D - 2A + BC$?

A. 4

B. 5

C. 6

D. 7

E. 8

**4.** (6 points) Find the average value of the function $y = 4x^3 + 2x - 1$ on the interval $[2, 4]$.

A. 113

B. 125

C. 129

D. 134

E. 149

**5.** (12 points) Find the volume of the solid $S$, where the base of $S$ is the region enclosed by the parabola $y = 1 - x^2$ and the $x$-axis. Cross-sections perpendicular to the $y$-axis are squares.

**6.** (12 points)

(a) Sketch a graph of the region bounded by curves $y = e^{-x}$, $y = -\frac{1}{2}x^2 + 2x + 1$, and $x = 3$. Label the points of intersection.

(b) Set up but do not evaluate an integral that represents the area bounded by the curves $y = e^{-x}$, $y = -\frac{1}{2}x^2 + 2x + 1$, and $x = 3$.

**7.** (12 points) Evaluate the integral.

$$\int \tan^7 x \sec^4 x \, dx$$

**8.** (16 points) Evaluate the integral.

$$\int \frac{(x+1)^3}{x^2+2x+10}\,dx$$

[Hint: Consider using long division.]

**9.** (12 points) If 8 J of work is needed to stretch a spring 4 m beyond its natural length, how much work is required to stretch the spring 5 m beyond its natural length?

**10.** (12 points)

(a) (4 points) Sketch a graph of the curves $y = 3x - x^2$ and $y = 2x$ on same set of axes and label the points of intersection.

(b) (8 points) Set up but do not compute an integral that represents the volume of the solid obtained by rotating about the $x$-axis the region bounded by the curves $y = 3x - x^2$ and $y = 2x$ using the method of cylindrical shells.