**1.** (6 points) Find a vector perpendicular to the plane containing the points $P(4,3,7)$, $Q(-3,0,1)$ and $R(2,-6,5)$.

A. $\langle 3,-25,9\rangle$
B. $\langle 57,-6,-30\rangle$
C. $\langle 6,17,18\rangle$
D. $\langle 14,27,12\rangle$
E. $\langle -48,-2,57\rangle$

$\vec{PQ} = \langle -3-4, 0-3, 1-7\rangle = \langle -7,-3,-6\rangle$

$\vec{PR} = \langle 2-4, -6-3, 5-7\rangle = \langle -2,-9,-2\rangle$

$$\vec{PQ}\times\vec{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -7 & -3 & -6 \\ -2 & -9 & -2 \end{vmatrix} = (6-54)\hat{i} - (14-12)\hat{j} + (63-6)\hat{k}$$

$= -48\hat{i} - 2\hat{j} + 57k$

$= \langle -48, -2, 57\rangle$

**2.** (6 points) Given that $|\mathbf{a}| = 4$, $|\mathbf{b}| = 7$ and the angle between $\mathbf{a}$ and $\mathbf{b}$ is $\frac{\pi}{6}$, what is $\mathbf{a}\cdot\mathbf{b}$?

A. 14
B. $14\sqrt{2}$
C. $14\sqrt{3}$
D. $14\sqrt{6}$
E. 28

$a\cdot b = |a||b|\cos\theta$

$= 4\cdot 7\cos\frac{\pi}{6}$

$= 28\cdot\frac{\sqrt{3}}{2}$

$= 14\sqrt{3}$

**3.** (6 points) The partial fraction decomposition of

$$\frac{3x^2 - x + 4}{x^4 - x^3 + 2x^2 - 2x}$$

has the form

$$\frac{A}{x} + \frac{B}{x-1} + \frac{Cx+D}{x^2+2}.$$

What is $3D - 2A + BC$?

A. 4

B. 5

C. 6

D. 7

E. 8

$3x^2 - x + 4 = A(x-1)(x^2+2) + Bx(x^2+2) + (Cx+D)x(x-1)$

$x=1$: $3 - 1 + 4 = 3B$

$\boxed{2 = B}$

$x=0$: $4 = -2A$

$\boxed{-2 = A}$

$(A+B+C)x^3 = 0$

$\Downarrow$

$2 - 2 + C = 0$

$\boxed{C = 0}$

$(2A + 2B - D)x = -x$

$\Rightarrow 2A + 2B - D = -1$

$\boxed{D = 1}$

$3D - 2A + BC$

$= 3 + 4$

$= \boxed{7}$

**4.** (6 points) Find the average value of the function $y = 4x^3 + 2x - 1$ on the interval $[2, 4]$.

A. 113

B. 125

C. 129

D. 134

E. 149

$$\frac{1}{4-2}\int_2^4 (4x^3 + 2x - 1)\,dx$$

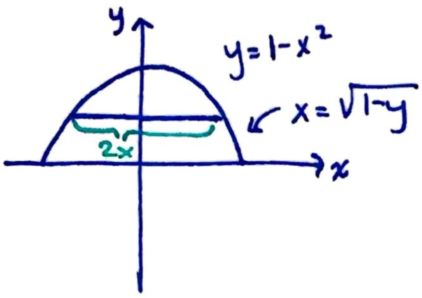
$$= \frac{1}{2}\int_2^4 (4x^3 + 2x - 1)\,dx$$

$$= \frac{1}{2}\left[x^4 + x^2 - x\right]_2^4$$

$$= \frac{1}{2}[268 - 18]$$

$$= 125$$

5. (12 points) Find the volume of the solid $S$, where the base of $S$ is the region enclosed by the parabola $y = 1 - x^2$ and the $x$-axis. Cross-sections perpendicular to the $y$-axis are squares.




$x^2 = 1-y$

$x = \pm\sqrt{1-y}$

$2x = 2\sqrt{1-y}$

So Area of cross section is

$(2\sqrt{1-y})^2 = 4(1-y) = 4-4y$

$V = \int_0^1 (4-4y)\,dy$

$= 4y - 2y^2 \Big|_0^1$

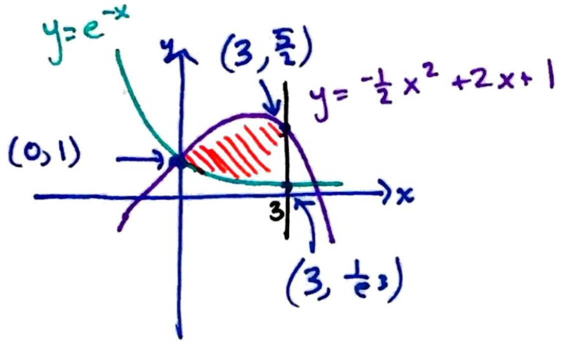
$= 4 - 2$

$= 2$

$\boxed{2}$

**6.** (12 points)

(a) Sketch a graph of the region bounded by curves $y = e^{-x}$, $y = -\frac{1}{2}x^2 + 2x + 1$, and $x = 3$. Label the points of intersection.




(b) Set up but do not evaluate an integral that represents the area bounded by the curves $y = e^{-x}$, $y = -\frac{1}{2}x^2 + 2x + 1$, and $x = 3$.

$$\boxed{\int_0^3 \left(-\frac{1}{2}x^2+2x+1 - e^{-x}\right) dx}$$

**7.** (12 points) Evaluate the integral.

$$\int \tan^7 x \sec^4 x \, dx$$

$= \int \tan^7 x \sec^2 x \sec^2 x \, dx$

$= \int \tan^7 x \,(1 + \tan^2 x) \sec^2 x \, dx$ $\quad u = \tan x$, $du = \sec^2 x \, dx$

$= \int u^7 (1 + u^2) \, du$

$= \int (u^7 + u^9) \, du$

$= \frac{1}{8} u^8 + \frac{1}{10} u^{10} + C$

$= \frac{1}{8} \tan^8 x + \frac{1}{10} \tan^{10} x + C$

$$\boxed{\frac{1}{8} \tan^8 x + \frac{1}{10} \tan^{10} x + C}$$

**8.** (16 points) Evaluate the integral.

$$I = \int \frac{(x+1)^3}{x^2+2x+10}\,dx$$

[Hint: Consider using long division.]

$$(x+1)^3 = x^3 + 3x^2 + 3x + 1$$

$$\begin{array}{r|l} & x + 1 \\ x^2+2x+10 & x^3 + 3x^2 + 3x + 1 \\ & -(x^3 + 2x^2 + 10x) \downarrow \\ \hline & x^2 - 7x + 1 \\ & -(x^2 + 2x + 10) \\ \hline & -9x - 9 \end{array}$$

$$I = \int \left[x + 1 - 9\left(\frac{x+1}{x^2+2x+10}\right)\right] dx$$

$$= \frac{1}{2}x^2 + x - 9\int \frac{x+1}{x^2+2x+10}\,dx$$

$u = x^2+2x+10$

$du = 2x+2\,dx$

$= 2(x+1)\,dx$

$$= \frac{1}{2}x^2 + x - \frac{9}{2}\int \frac{du}{u}$$

$$= \frac{1}{2}x^2 + x - \frac{9}{2}\ln|u| + C$$

$$= \frac{1}{2}x^2 + x - \frac{9}{2}\ln|\underbrace{x^2+2x+10}_{\text{always} > 0.}| + C$$

$$\boxed{\frac{1}{2}x^2 + x - \frac{9}{2}\ln(x^2+2x+10) + C}$$

**9.** (12 points) If 8 J of work is needed to stretch a spring 4 m beyond its natural length, how much work is required to stretch the spring 5 m beyond its natural length?

$$8 = \int_0^4 kx\,dx$$

$$8 = \frac{1}{2}kx^2 \Big|_0^4$$

$$8 = \frac{1}{2}k(16)$$

$$\boxed{k = 1}$$

$$\int_0^5 x\,dx$$

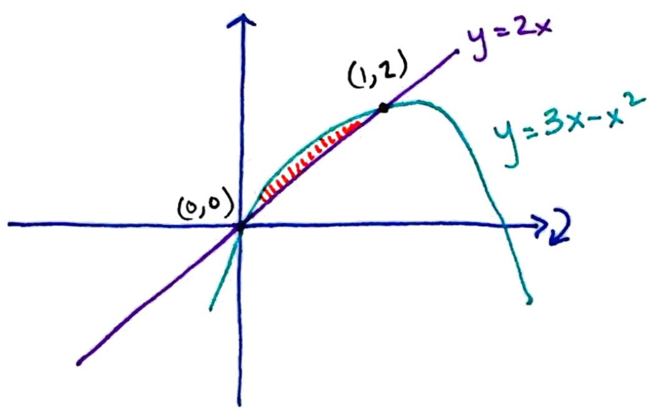
$$= \frac{1}{2}x^2 \Big|_0^5$$

$$= \frac{25}{2}$$

$$\boxed{\frac{25}{2}\text{ J}}$$

**10.** (12 points)

(a) (4 points) Sketch a graph of the curves $y = 3x - x^2$ and $y = 2x$ on same set of axes and label the points of intersection.




$3x - x^2 = 2x$

$0 = x^2 + 2x - 3x$

$= x^2 - x$

$= x(x-1)$

$x = 0, 1$

(b) (8 points) Set up but do not compute an integral that represents the volume of the solid obtained by rotating about the $y$-axis the region bounded by the curves $y = 3x - x^2$ and $y = 2x$ using the method of cylindrical shells.

radius: $x$

height: $(3x - x^2) - 2x = x - x^2$

$$\boxed{\int_0^1 2\pi x (x - x^2)\, dx}$$