MA 16200 **Midterm Exam 3** **Summer 2017**

Name: ______________________________

ID number: ______________________________

Instructions:

1. This is a one-hour exam.
2. There are 10 problems on this exam.
3. No books, notes, or calculators are allowed.
4. Please turn off your cell phone.
5. Circle one and only one choice for each multiple-choice problem. No partial credit will be given for multiple-choice problems.
6. Show all relevant work on non-multiple-choice problems. Partial credit will be given for steps leading to the correct solutions.
7. You may use a writing utensil, your own brain and the paper provided in this exam. Use of any other persons or resources will be considered cheating and will be reported to the Office of the Dean of Students.

I agree to abide by the instructions above:

Signature: ______________________________

Useful trig formulas:

$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ $\qquad$ $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$ $\qquad$ $\sin x \cos x = \frac{1}{2}\sin 2x$

Page	Score	Points Possible
2		14
3		14
4		12
5		12
6		12
7		12
8		12
Total		100

1. (7 points) Suppose $\sum a_n$ and $\sum b_n$ are series, with $b_n > 0$ for all n. Which of the following must be true?

I. If $\sum a_n$ converges, then $\sum |a_n|$ also converges.

II. If $\sum b_n$ converges, then $\sum (-1)^{n-1} b_n$ also converges.

III. If $\lim\limits_{n\to\infty} b_n = 0$, then $\sum (-1)^{n-1} b_n$ converges.

A. I only.

B. II only.

C. III only.

D. II and III only.

E. I, II and III.

2. (7 points) Suppose a series $\sum a_n$ is defined recursively as follows

$$a_1 = 1, \quad a_{n+1} = \frac{2 + \cos n}{\sqrt{n}} a_n.$$

Which of the following statements is true?

A. The series converges absolutely.

B. The series converges conditionally.

C. The series diverges.

D. There is not enough information to test for convergence.

E. This answer is incorrect.

3. (7 points) Evaluate the sum

$$1 - \ln 2 + \frac{(\ln 2)^2}{2!} - \frac{(\ln 2)^3}{3!} + \cdots$$

A. $-\ln 2$

B. $\ln 2$

C. $\frac{1}{2}$

D. 2

E. The series does not converge.

4. (7 points) Suppose a parametric curve is given by functions $x(t)$, $y(t)$ with

$$\frac{dx}{dt} = 1 - \frac{1}{t} \quad \text{and} \quad \frac{dy}{dt} = 1 + \frac{1}{t}.$$

Find $\frac{d^2y}{dy^2}$.

A. -1

B. $\dfrac{-1/t^2}{1 - 1/t}$

C. $\dfrac{2}{t^2(t-1)}$

D. $\dfrac{-2t}{(t-1)^3}$

E. More information is needed to be able to compute $\frac{d^2y}{dx^2}$.

5. (12 points) Determine whether the following series converge. Fully justify your response.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{n \cos n\pi}{2^n}$

(b) $\displaystyle\sum_{n=0}^{\infty} ne^{-n}$

6. (12 points) Determine whether the following series are conditionally convergent, absolutely convergent or divergent. Fully justify your response.

(a) $\displaystyle\sum_{k=2}^{\infty} \frac{(-1)^{k-1}}{(\ln k)^k}$

(b) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{n^2}$

7. (12 points) Find the exact length of the curve given by

$$x(t) = 1 + 2t, \quad y(t) = e^t + e^{-t}$$

for t in the interval $[0, \ln 7]$.

8. (12 points) Compute the following limit.

$$\lim_{x\to 0} \frac{\cos(x^2) + \frac{1}{2}x^4 - 1}{x^8}$$

9. (12 points) Let

$$f(x) = \sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2} x^n.$$

What is the radius of convergence R of this power series? What is the radius of convergence R' of the power series for $f'(x)$? For partial credit, you may write how R' is related to R.

10. (12 points) Find the Maclaurin series for the function

$$f(x) = x^3 \tan^{-1}(x^2).$$

What is the radius of convergence?