7.6 Integration using tables

Ex 1) $\int \frac{\cos x}{\sin^2 x - 9}\,dx$ $\qquad u = \sin x$, $du = \cos x\,dx$

$= \int \frac{du}{u^2 - 9} \overset{20}{=} \frac{1}{6}\ln\left|\frac{u-3}{u+3}\right| + C$

$= \boxed{\frac{1}{6}\ln\left|\frac{\sin x - 3}{\sin x + 3}\right| + C}$

Ex2

$I = \int y\sqrt{6+4y-4y^2}\,dy$

$6 + 4y - 4y^2 = 6 - 4(-y+y^2) = 1 + 6 - 4(y^2 - y + \frac{1}{4})$

$= 7 - 4(y - \frac{1}{2})^2 = 7 - (2y-1)^2$

Now $I = \int y\sqrt{7-(2y-1)^2}\,dy$ $\qquad u = 2y-1 \Rightarrow y = \frac{1}{2}(u+1)$, $du = 2\,dy$

$= \frac{1}{4}\int (u+1)\sqrt{7-u^2}\,du$

$= \frac{1}{4}\left[\int u\sqrt{7-u^2}\,du + \int\sqrt{7-u^2}\,du\right]$

$= \frac{1}{4}\left[-\frac{1}{2}\int s^{1/2}\,ds + \int\sqrt{7-u^2}\,du\right]$ #30

$= \frac{1}{4}\left[-\frac{1}{2}\cdot\frac{2}{3}s^{3/2} + \frac{u}{2}\sqrt{7-u^2} + \frac{7}{2}\sin^{-1}\frac{u}{\sqrt{7}}\right] + C$

$= \boxed{-\frac{1}{12}(6+4y-4y^2)^{3/2} + \frac{2y-1}{8}\sqrt{6+4y-4y^2} + \frac{7}{2}\sin^{-1}\left(\frac{2y-1}{\sqrt{7}}\right) + C}$

Not always possible to find anti-derivative.

Ex $y = e^{x^2}$.

Approximating integrals

Already know one way: Riemann sums, using Left, Right endpts, and midpoints.

Recall Midpoint rule

$$\int_a^b f(x)\,dx \approx \Delta x\left[f(\bar{x}_1) + f(\bar{x}_2) + \cdots + f(\bar{x}_n)\right],$$

where $\Delta x = \frac{b-a}{n}$, $\bar{x}_i = \frac{1}{2}(x_{i-1} + x_i)$ = midpt of $[x_{i-1}, x_i]$.

Two new methods:

Trapezoidal Rule This is an average of left- and right-hand sums.

$$\int_a^b f(x)\,dx \approx \frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f(x_n)\right]$$

where $\Delta x = \frac{b-a}{n}$, $x_i = a + i\Delta x$.

Error bounds:

Suppose $|f''(x)| \leq K$ for $a \leq x \leq b$ for some K. If E_T and E_M are the errors in the Trapezoidal and Midpoint Rules, resp, then

$$|E_T| \leq \frac{K(b-a)^3}{12n^2} \quad \text{and} \quad |E_M| \leq \frac{K(b-a)^3}{24n^2}.$$

Simpson's Rule Uses parabolas instead of lines to approximate curves. Derivation on p. 520 in Stewart.

$$\int_a^b f(x)\,dx \approx \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\right]$$

where n is even and $\Delta x = (b-a)/n$.

Ex Approximate $\int_1^2 \underbrace{\sqrt{x^3-1}}_{f(x)}\,dx$ using $n=10$ for (a) Trapezoidal, (b) Midpt and (c) Simpson's Rule.

(a) $\Delta x = \frac{2-1}{10} = \frac{1}{10}$

$$T_{10} = \frac{1}{20}\left[f(1) + 2f(1.1) + 2f(1.2) + 2f(1.3) + 2f(1.4) + \cdots + 2f(1.9) + f(2)\right]$$

(b) $\Delta x = \frac{2-1}{10} = \frac{1}{10}$

$$M_{10} = \frac{1}{10}\left[f\left(\frac{1+1.1}{2}\right) + f\left(\frac{1.1+1.2}{2}\right) + \cdots + f\left(\frac{1.9+2}{2}\right)\right]$$

$$= \frac{1}{10}\left(f(1.05) + f(1.15) + \cdots + f(1.95)\right]$$

(c) $\Delta x = \frac{1}{10}$

$$S_{10} = \frac{1}{30}\left[f(1) + 4f(1.1) + 2f(1.2) + \cdots + 4f(1.8) + 2f(1.9) + f(2)\right]$$