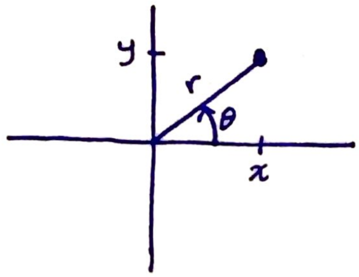
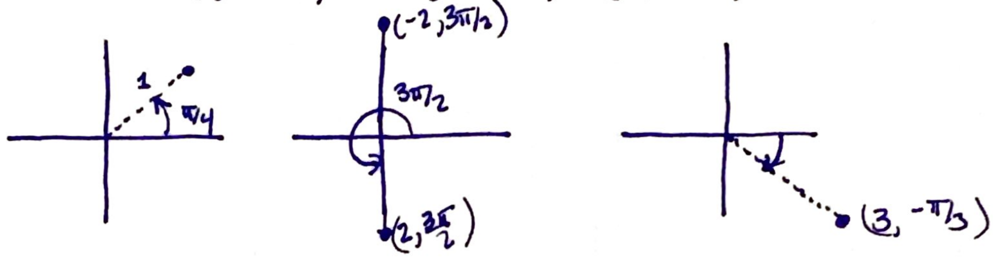
10.3 Polar Coordinates



We can represent any point in Cartesian plane with an angle θ and radius r. Positive angles are measured counterclockwise.

Ex 1 Plot $(1, \pi/4)$; $(-2, 3\pi/2)$; $(3, -\pi/3)$



Note: If $r<0$, the point is in the opposite direction. That is, $(-r, \theta) = (r, \theta + \pi)$

Ex 2 Find ~~polar~~ Cartesian coords ~~where $r \geq 0$, $0 \leq \theta < 2\pi$~~

(a) ~~$(4, 4)$~~
$(2, 3\pi/2)$

$x = r\cos\theta$, $\quad y = r\sin\theta$

$x = 2\cos\left(\frac{3\pi}{2}\right)$, $\quad y = 2\sin\left(\frac{3\pi}{2}\right)$

$x = 0$, $\quad y = -2$

$(0, -2)$

(b) $(\sqrt{2}, \pi/4)$

$$\left.\begin{aligned} x &= \sqrt{2}\cos(\pi/4) = \sqrt{2}\cdot\tfrac{1}{\sqrt{2}} \\ y &= \sqrt{2}\sin(\pi/4) = \sqrt{2}\cdot\tfrac{1}{\sqrt{2}} \end{aligned}\right\} = (1, 1)$$

Since $x = r\cos\theta$, $y = r\sin\theta$, we know

$$x^2 + y^2 = r^2(\cos^2\theta + \sin^2\theta) = r^2$$

$$\frac{y}{x} = \frac{r\sin\theta}{r\cos\theta} = \tan\theta$$

We can use this to convert from cartesian to polar

Ex3

$(3, 3\sqrt{3})$

$$r^2 = 3^2 + (3\sqrt{3})^2 = 9 + 27 = 36 \Rightarrow r = 6$$

$$\frac{y}{x} = \tan\theta = \frac{3\sqrt{3}}{3} = \sqrt{3} \Rightarrow \theta = \pi/3$$

Polar coords: $(6, \pi/3)$

Ex 4 $(-4, 4)$

$$r^2 = (-4)^2 + 4^2 = 32 \Rightarrow r = 4\sqrt{2}$$

$$\frac{y}{x} = \tan\theta = \frac{4}{-4} = -1 \Rightarrow \theta = \frac{3\pi}{4}$$

$(r, \theta) = (4\sqrt{2}, 3\pi/4)$

Remark When converting from Cartesian to polar, we need to take note of what quadrant we're in to find θ.

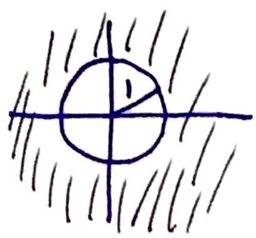
Ex 5 Sketch the region

a) $r \geq 1$

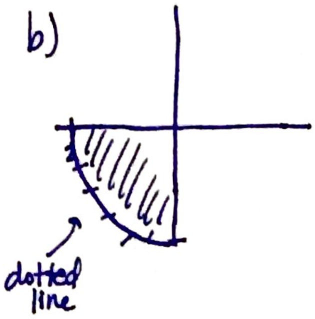
b) $0 \leq r < 2$, $\pi \leq \theta \leq 3\pi/2$

c) $2 < r < 3$ $\frac{5\pi}{3} \leq \theta \leq 7\pi/3$

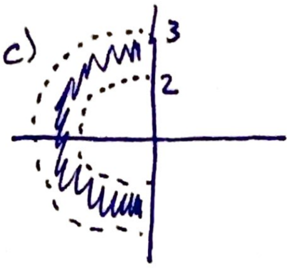
a)



b)



c)



Ex 6 Find cartesian eqn, identify curve.

a) $r = 4\sec\theta$, b) $r^2 = 5$, c) $\theta = \pi/3$, d) $r^2 \sin 2\theta = 1$

a) $r = 4\sec\theta \Rightarrow r\cos\theta = 4 \Rightarrow x = 4$ (vertical line)

b) $r^2 = 5 \Rightarrow r^2(\cos^2\theta + \sin^2\theta) = (r\cos\theta)^2 + (r\sin\theta)^2 = 5$ (circle of radius $\sqrt{5}$)

c) $\theta = \pi/3 \Rightarrow \frac{y}{x} = \tan\frac{\pi}{3} \Rightarrow \frac{y}{x} = \sqrt{3} \Rightarrow y = \sqrt{3}\,x$ (line)

d) $r^2 \sin 2\theta = 1 \Leftrightarrow r^2(2\sin\theta\cos\theta) = 1 \Leftrightarrow (r\cos\theta)(r\sin\theta) = \frac{1}{2}$

$\Leftrightarrow xy = 1/2$ (hyperbola)

Ex 7 Find a polar eqn

$4y^2 = x$

$4r^2\sin^2\theta = r\cos\theta$

$4r^2\sin^2\theta - r\cos\theta = 0$

Ex 8 $x^2 - y^2 = 4$

$r^2\cos^2\theta - r^2\sin^2\theta = 4$

$r^2(\cos^2\theta - \sin^2\theta) = 4$

$r^2 = 4/(\cos^2\theta - \sin^2\theta) = 4\sec 2\theta$