MATH 525, Practice exam, Spring 2001.

There will be one multiple choice problem (True/False), with 10 questions, worth 1 point each, and 4 “partial credit” problems, with maximum 10 points each.

1. True or false questions.

a) $\mathrm{Arg}\, z\}$ is a harmonic function in its domain.

b) If u and v are harmonic functions, then $u + iv$ is analytic.

c) If f is an analytic function then $\exp f$ is also analytic.
d) The equation $e^z = a$ has infinitely many solutions for every complex a.

e) There is a branch of the inverse to the Joukovskii function in the upper half-plane.

f) The set $\{z : |e^z - 1| < 10$ is connected.

g) The function of a complex variable $f(z) = (\sin z)/z$ has a limit when $z \to 0$.

h) If f is analytic, and $\overline{f(\overline{z})}$ is also analytic, then $f = \mathrm{const}$.

i) The equation $z^9 9 = -1$ has exactly 99 complex solutions, only one of them real.

j) All analytic functions are continuous.

2. Find a harmonic conjugate to $u(x, y) = e^x(x \sin y + y \cos x)$.

3. Let f be a branch of $\log(2z - 1)$ in the region which is obtained from the complex plane by deleting the ray $\{z = -1 + it : 0 \leq t < \infty\}$. Suppose that $f(0) = 3\pi i$. Find $f(-2)$.

4. For every integer m, evaluate the integral

$$\int_\gamma \overline{z}^m dz,$$

where γ is a circle centered at the origin, traced anticlockwise.

5. Determine the set, consisting of the points z, where the values of Joukowski's function $f(z) = (z + z^{-1})/2$ are real. Sketch this set.