Math 520, Spring 2007, Midterm exam
(No books, no calculators)

NAME:

1. Solve the heat equation for a rod

$$u_t = u_{xx}, \quad 0 \le x \le \pi, \quad t \ge 0$$

with insulated ends, $u_x(0,t) = u_x(\pi,t) = 0,\ t \ge 0$ and the initial temperature $u(x,0) = \sin x$. How much time (approximately) is needed for the initial temperature to level, so that its variation along the segment $[0,\pi]$ does not exceed 1%?

2. Suppose that f is a smooth 2π-periodic function, and

$$f(t) = \sum_{-\infty}^{\infty} c_n e^{int}$$

its Fourier expansion. How can one find out from the sequence (c_n) that:

a) f is real

b) f is even

c) f has π as a period.

3. Consider the Sturm-Liouville problem

$$y'' + \lambda y = 0, \quad y(0) = 0, \quad y(1) + y'(1) = 0.$$

a) Is this problem self-adjoint? Explain your answer.

b) How many eigenvalues λ_j satisfy $0 < \lambda_j < 4\pi$?

4. Given that the Fourier sine series for $f(x) = x(\pi - x),\ 0 \le x \le \pi$ is

$$f(x) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1 - (-1)^n}{n^3} \sin nx,$$

find the solution of the inhomogeneous heat equation

$$u_t = 2u_{xx} + e^{-t}, \quad u(0.t) = u(\pi, t) = 0, \quad u(x, 0) = f(x).$$

5. Expand the function $x/|x|$ into a Fourier series on $(-\pi, \pi)$ and use the Parseval identity to find the sum of the series

$$\sum_{n=0}^{\infty} \frac{1}{(2n+1)^2}.$$