What are the eigenvalues of

$$vv^T + ww^T, \quad v, w \in R^n?$$

Ans. Orthogonalize: $v = au_1,\ w = bu_1 + cu_2$. Complete u_1, u_2 to an orthonormal basis. Then in this basis the linear operator will have matrix

$$\begin{pmatrix} a^2 + b^2 & bc \\ bc & c^2 \end{pmatrix},$$

extended by zeros.