## MA262 — PRACTICE PROBLEMS FOR EXAM II — SPRING 2022
## BASED ON OLD MA262 EXAMS WHICH CAN BE FOUND AT
https://www.math.purdue.edu/academic/courses/oldexams.php?course=MA26200

This is a collection of problems from old exams, the textbook and some other exercises. We have been using a new textbook book since the fall 2020 and some old exam problems do not match the notation or the content.

**1.** Review of Algebra find all real and complex roots of the following polynomial equations

I. $z^4 + 16 = 0$
II. $z^4 - 16 = 0$
III. $z^3 + 81 = 0$
IV. $z^3 - 81 = 0$
V. $z^2 + 2z + 5 = 0$
VI. $z^2 + 2z + 3 = 0$

**2.** Find the general solutions of the differential equations corresponding to the above polynomial equations

I. $y^{(4)} + 16y = 0$
II. $y^{(4)} - 16y = 0$
III. $y^{(3)} + 81y = 0$
IV. $y^{(3)} - 81y = 0$
V. $y'' + 2y' + 5y = 0$
VI. $y'' + 2y' + 3y = 0$

**3.** Consider the vectors

$$v_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 3 \\ 1 \\ 7 \\ 1 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 5 \\ -3 \\ 9 \\ 1 \end{bmatrix} \quad v_4 = \begin{bmatrix} -2 \\ 4 \\ 2 \\ 8 \end{bmatrix}$$

The dimension of the vector space $\text{span}\{v_1, v_1, v_3, v_4\}$ is equal to

A. 1

B. 2

C. 3

D. 4

E. 5

**4.** Which of the following are subspaces of a given vector space?

I. $S$ is the set of all $(x, y) \in \mathbb{R}^2$ such that $x + y \geq 0$.

II. $S$ is the set of all functions $f(x)$ defined on $(-2, 2)$ such that $f(1) + f(-1) = 0$.

III. $S$ is the set of all $(x_1, x_2, x_3, x_4)$ in $\mathbb{R}^4$ such that $x_1 + x_4 + 9x_3 = 0$

IV. $S$ is the set of all polynomials $P(x)$ such that $P'(x) + P(x) = 1$.

A. Only I

B. Only II and III

C. Only IV

D. Only II and IV

E. Only II, III and IV

**5.** Let $\mathcal{P}_4$ denote the space of polynomials of degree less than or equal to four. Let $\mathcal{V}$ be the subspace consisting of polynomials $p(x)$ in $\mathcal{P}_4$ such that $p(0) = 0$. We can say that

A. The dimension of $\mathcal{P}_4$ is equal to four and the dimension of $\mathcal{V}$ is equal to three

B. The dimension of $\mathcal{P}_4$ is equal to three and the dimension of $\mathcal{V}$ is equal to two

C. The dimension of $\mathcal{P}_4$ is equal to five and the dimension of $\mathcal{V}$ is equal to four

D. The dimension of $\mathcal{P}_4$ is equal to five and the dimension of $\mathcal{V}$ is equal to three

E. The dimension of $\mathcal{P}_4$ is equal to two and the dimension of $\mathcal{V}$ is equal to one

**6.** Find all values of $k$ such that the vectors $v_1 = (1, -1, 0)$ $v_2 = (1, 2, 2)$ and $v_3 = (0, 3, k)$ form a basis for $\mathbb{R}^3$.

A. $k = 1$

B. $k = 2$

C. $k \neq 1$

D. $k \neq 2$

E. $k \neq 3$

**7.** Let

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \text{ and } B = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix}.$$

If the determinant of $A$ is equal to five and the determinant of $B$ is equal to 6 the determinant of the matrix

$$C = \begin{pmatrix} a_{11} & 2a_{12} & a_{13} \\ a_{21} & 2a_{22} & a_{23} \\ a_{31} & 2a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} 3b_{11} & b_{12} + 2b_{11} & b_{13} \\ 3b_{21} & b_{22} + 2b_{21} & b_{23} \\ 3b_{31} & b_{32} + 2b_{31} & b_{33} \end{pmatrix}$$

is equal to

A. 180

B. 100

C. 150

D. 200

E. 300

**8.** If $y(x)$ is the solution of

$$y'' - 2y' + y = 0,$$
$$y(0) = 1, \quad y'(0) = -1$$

then $y(\frac{1}{2})$ is equal to

A. $0$

B. $e^{\frac{1}{2}}$

C. $2e^{\frac{1}{2}}$

D. $\dfrac{1}{2}e^{\frac{1}{2}}$

E. $-3e^{\frac{1}{2}}$

**9.** Find linearly independent solutions of $x^2y''(x)+4xy'(x)+2y(x) = 0$ $x > 0$ of the form $y(x) = x^r$ and conclude that the general solution of this equation, for $x > 0$, is of the form

A. $y(x) = C_1x^{-1} + C_2x^{-2}$

B. $y(x) = C_1x^{-2} + C_2x^{-3}$

C. $y(x) = C_1x^{-2} + C_2x^{-4}$

D. $y(x) = C_1x^{-3} + C_2x^{-5}$

E. $y(x) = C_1 + C_2x^{-3}$

**10.** Let $y(x)$ satisfy

$$x^2y''(x) - 2xy'(x) + 2y(x) = 0$$
$$y(1) = 2, \quad y'(1) = 3.$$

Then $y(2)$ is equal to (Hint: use the method of the previous problem.)

A. 4

B. 5

C. 6

D. 7

E. 8

**11.** The constants $a, b, c, d$ are real numbers and the solutions of the polynomial equation $p(r) = ar^3 + br^2 + cr + d = 0$, are given by $r_1 = 1 + 2i$, $r_2 = 1 - 2i$ and $r_3 = 4$. We can say that the general solution of the equation $ay^{(3)} + by^{(2)} + cy' + dy = 0$ is given by

A. $y(x) = C_1e^{4x} + C_2e^x \cos(2x) + C_3e^x \sin(2x)$

B. $y(x) = C_1e^{4x} + C_2e^{2x} \cos(2x) + C_3e^{2x} \sin(2x)$

C. $y(x) = C_1e^{4x} + C_2e^{4x} \cos(x) + C_3e^{4x} \sin(x)$

D. $y(x) = C_1e^{4x} + C_2e^{2x} \cos(x) + C_3e^{2x} \sin(x)$

E. $y(x) = C_1e^{4x} + C_2e^x \cos(x) + C_3 \sin(x)$

**12.** We know that the characteristic polynomial of a certain homogeneous differential equation is given by

$$p(r) = (r^2 - 1)^2(r^2 + 4)^2.$$

We can say that the general solution of the differential equation is given by

A. $C_1e^x + C_2xe^x + C_3e^{-x} + C_4xe^{-x} + C_5\cos(2x) + C_6x\cos(2x) + C_7\sin(2x) + C_8x\sin(2x)$

B. $C_1e^x + C_2xe^x + C_3e^{-x} + C_4xe^{-x} + C_5(1+x)\cos(2x) + C_6(1+x)\sin(2x)$

C. $C_1e^x+C_2xe^x+C_3x^2e^x+C_4(1+x)e^{-x}+C_5\cos(2x)+C_6x\cos(2x)+C_7x^2\cos(2x)+C_8x\sin(2x)$

D. $C_1e^x + C_2e^{-x} + C_3\cos(2x) + C_4\sin(2x)$

E. $C_1e^x + C_2e^{-x} + C_3\cos(2x) + C_4x\cos(2x) + C_5\sin(2x) + C_6x\sin(2x)$

F. $C_1e^x + C_2xe^x + C_3e^{-x} + C_4xe^{-x} + C_5\cos(2x) + C_6x\sin(2x) + C_7x^2\sin(2x)$

**13.** The Wronskian of the functions $\{x, \sin x, \cos x\}$ (in this order) is equal to

A. $x$

B. $-x$

C. $x\sin x + \cos x$

D. $\sin x\cos x$

E. $x(\cos^2 x - \sin^2 x)$

The next page has two additional problems from exam 2, fall 2019.

6. The following system of linear equations has an infinite number of solutions:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 &= 0, \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 &= 1, \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 &= 0, \end{aligned}$$

In this case the matrix $A = (a_{ij}) \in R^{3\times 3}$ satisfies the condition

A. $A$ is defective

B. $A$ is nonsingular

C. The homogeneous systems $A\mathbf{x} = 0$ has exactly one solution

D. The homogeneous system $A\mathbf{x} = 0$ has more than one solution

E. None of the above

**7.** Let $A$ be a $4 \times 4$ matrix. If $\mathbf{p} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{bmatrix}$ and $\mathbf{q} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$ are solutions of the system of linear equations $A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 3 \end{bmatrix}$, then which of the following is also a solution?

A. $\mathbf{p} - \mathbf{q}$

B. $\mathbf{q} - 2\mathbf{p}$

C. $2\mathbf{p} - \mathbf{q}$

D. All of the above

E. None of the above

8. The entry $b_{32}$ of the matrix $A^{-1} = (b_{ij})$ inverse to $A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 4 & 1 \\ 2 & -4 & 0 \end{bmatrix}$ is equal to

A. $-\frac{1}{2}$

B. $\frac{3}{2}$

C. $\frac{2}{3}$

D. $2$

E. $4$

**10.** $A$ is an $m \times n$ matrix and $\mathbf{b}$ is an $m \times 1$ vector. The equation $A\mathbf{x} = \mathbf{b}$ has **infinitely many** solutions. Consider the following statements:

(i) $m \leq n$

(ii) $n \leq m$

(iii) the rank of $A = n$

(iv) the rank of $A < n$

(v) $\det A = 0$

Which **must** be true?

A. only (i) and (v)

B. only (iv)

C. only (v)

D. only (iii) and (v)

E. None of the statements has to be true

15. Let $S$ denote the set of all polynomials of degree less than or equal to 4. What is the dimension of $S$?

A. 1

B. 2

C. 3

D. 4

E. 5

(14) If

$$A = \begin{bmatrix} 1 & -2 & 0 & 4 \\ 3 & 1 & 1 & 0 \\ -1 & -5 & -1 & 8 \\ 3 & 8 & 2 & -12 \end{bmatrix}$$

then the rank of $A$ is

(A) 0

(B) 1

(C) 2

(D) 3

(E) 4

(18) Assume that the $4 \times 4$ matrix $A$ is row equivalent to the matrix $B$, where

$$B = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Which of the following statements is true?

(A) $B$ is not the reduced row echelon form of $A$.

(B) $\det(A) \neq 0$.

(C) The null space of $A$ has dimension 3.

(D) The column space of $A$ has dimension 2.

(E) If $A\vec{x} = \vec{b}$ is consistent for some $\vec{b} \neq \vec{0}$, then it has infinitely many solutions.

**11.** If the dimension of the nullspace of a $5 \times 6$ matrix $A$ is 4, what are the rank of $A$ and the dimension of the column space of $A$?

A. rank($A$)=3, dim[Col($A$)]=3.
B. rank($A$)=1, dim[Col($A$)]=2.
C. rank($A$)=2, dim[Col($A$)]=1.
D. rank($A$)=1, dim[Col($A$)]=1.
E. rank($A$)=2, dim[Col($A$)]=2.

**4.** Which of the following statements about all $5 \times 5$ matrices is **true**?

A. $\det(A+B) = \det A + \det B$

B. $\det A^T = -\det(A)$.

C. $AB = 0$ implies $A = 0$ or $B = 0$.

D. If $\det A = 0$ then some two rows are proportional.

E. $\det(-A) = -\det(A)$.

**6.** Consider the real vector space $\mathbb{M}_2$ of all real $2 \times 2$ matrices. Let $B$ be a fixed matrix in $\mathbb{M}_2$. Which of the following sets is **not** a subspace of $\mathbb{M}_2$?

A. The set of all the matrices, $A$ in $\mathbb{M}_2$ such that $AB = BA$.

B. The set of all the matrices, $A$ in $\mathbb{M}_2$ such that $AB = O$, where $O$ is the zero matrix in $\mathbb{M}_2$.

C. The set of all the matrices, $A$ in $\mathbb{M}_2$ such that $A^2 = O$, where $O$ is the zero matrix in $\mathbb{M}_2$.

D. All the upper triangular matrices in $\mathbb{M}_2$.

E. All the symmetric matrices in $\mathbb{M}_2$.

**7.** What is the dimension of the space span $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$?

A. 0

B. 1

C. 2

D. 3

E. 4

**8.** Let $A = \begin{pmatrix} 2 & -1 & -2 \\ -4 & 2 & 4 \\ -8 & 4 & 8 \end{pmatrix}$. Then the following is a basis of the nullspace of $A$.

A. $\{[2\ -4\ -8]^T, [-1\ 2\ 4]^T, [-2\ 4\ 8]^T\}$

B. $\{[1\ 0\ 1]^T, [1\ 2\ 0]^T\}$

C. $\{[1\ 0\ 1]^T, [1\ 2\ 0]^T, [3\ 2\ 2]^T\}$

D. $\{[1\ 0\ 1]^T\}$

E. $\{[0\ 2\ 1]^T\}$

**9.** Let $A$ be an $m \times n$ matrix. Then the linear system $A\mathbf{x} = \mathbf{b}$ has a solution for any $m \times 1$ matrix $\mathbf{b}$ if and only if

A. $m = n$.

B. Nullity($A$)$= m$.

C. Rank($A$)+Nullity($A$)$= n$.

D. $A = I$ the identity matrix.

E. Rank($A$)$= m$.

**7.** Find the general solution to the differential equation

$$y^{(4)} - 8y'' + 16y = 0.$$

A. $y = c_1 e^{2x} + c_2 e^{-2x}$

B. $y = c_1 x e^{2x} + c_2 x e^{-2x}$

C. $y = c_1 e^{2x} + c_2 e^{-2x} + c_3 x e^{2x} + c_4 x e^{-2x}$

D. $y = c_1 x e^{2x} + c_2 x e^{-2x} + c_3 x^2 e^{2x} + c_4 x^2 e^{-2x}$

E. $y = c_1 \cos 2x + c_2 \sin 2x + c_3 x \cos 2x + c_4 x \sin 2x$

**8.** Let $y(x)$ satisfy

$$y'' + 9y' + 18 = 0, \quad y(0) = 0, \quad y'(0) = 3.$$

Then $y(\frac{1}{3}\log 3) =?$

A. $y(\frac{1}{3}\log 3) = \frac{2}{9}$

B. $y(\frac{1}{3}\log 3) = \frac{2}{3}$

C. $y(\frac{1}{3}\log 3) = \frac{3}{4}$

D. $y(\frac{1}{3}\log 3) = \frac{1}{3}$

E. $y(\frac{1}{3}\log 3) = \frac{1}{4}$

**11.** Find the solution to the initial value problem

$$y'' + 2y' + 2y = 0, \quad y(0) = 1, \quad y'(0) = 0.$$

A. $y(t) = e^t + e^{-t}$
B. $y(t) = \cos t + \sin t$
C. $y(t) = e^t \cos t + e^t \sin t$
D. $y(t) = e^{-t} \cos t - e^{-t} \sin t$
E. $y(t) = e^{-t} \cos t + e^{-t} \sin t$

**12.** Consider the differential equation

$$y'' - 6y' + 9y = 0.$$

Which one of the following statements is true?

A. The equation has only one linearly independent solution $e^{3t}$
B. The equation has only one linearly independent solution $te^{3t}$
C. The equation has two linearly independent solutions $e^{3t}$ and $te^{3t}$
D. The equation has two linearly independent solutions $t$ and $e^{3t}$
E. The equation has two linearly independent solutions $t^3$ and $e^{3t}$