Kelvin's estimate of the age of the Earth

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The idea to use heat equation to estimate the age of the Earth was one of the principal motivations of Fourier's work on the heat equation.

It was known for long time, that the temperature increases with depth. The natural explanation of this fact was that the Earth was hot at the time of its formation, and since then it just cools down from the surface. Assuming some initial temperature, say the melting temperature of the rocks, and using the measured value of the gradient at the surface, one can estimate the age of the earth, that is the time passed since the rocks solidified.

The following computation is due to William Thomson (Lord Kelvin).

We use flat Earth approximation. x is the depth from the surface.

$$u_t = ku_{xx},$$

$$u(0,t) = 0,$$

$$u(x,0) = \left\{ \begin{array}{ll} T, & x > 0, \\ 0, & x < 0. \end{array} \right\}.$$

Here we use Kelvin's data: $T = 3900^\circ$ is the initial temperature, and $k = 1.2 \times 10^{-2} (cm^2/sec)$. The current gradient is $\beta = 3.6 \times 10^{-4} (^\circ/cm)$. We extend u as an odd function, to keep the boundary condition 0. Then $u + T$ will have the initial condition 0 on the negative semi-axis and $2T$ on the positive, so We have

$$u(x,t) = \frac{T}{\sqrt{\pi k t}} \int_0^\infty e^{-(x-y)^2/(4kt)} dy - T.$$

$$u_x(0,t) = \frac{T}{\sqrt{\pi k t}}.$$

Therefore

$$t = \frac{1}{\pi k}(T/\beta)^2.$$

Using our data and the number 3.2×10^7 seconds in a year, we obtain $t = 9.2 \times 10^7$ years, that is about 92 million years.

Geologists and evolutionary biologists objected this estimation as too low for their theories. Kelvin insisted. For this reason, people considered him anti-evolutionist and reactionary (see, for example, Wikipedia article about him).

The computation is correct, of course, for the given assumptions. Kelvin died in 1907. Radioactivity was discovered in 1896. It is radioactivity which is responsible for a large portion of the heat inside the Earth.

References

[1] William Thomson, On the secular cooling of the Earth, Trans. Royal Soc. Edinburgh, XXIII, (1862) 157–169.