Cellar theory

January 26, 2012

In class we solved the differential equation

$$u_t = ku_{xx}$$

for $x \geq 0$ and $-\infty < t < \infty$ with a periodic boundary condition at the end $x = 0$ and the condition that the solution is bounded at $+\infty$.

If the boundary condition is

$$u(0, t) = \cos \omega t,$$

then we obtained the solution in the form

$$u(x, t) = \exp\left(-x\sqrt{\frac{\omega}{2k}}\right) \cos\left(\omega t - x\sqrt{\frac{\omega}{2k}}\right).$$

For a typical termal diffusivity of soil $k = 0.2 \times 10^{-6} m^2/sec$, and period of the oscillation 1 year, find at what depth x, the shift of the seasons will be $1/2$ of a year.

By what factor the amplitude of the oscillations is smaller at this depth than the amplitude on the surface?

This problem explains, by the way, why one can keep ice in a cellar in summer in the places where the average yearly temperature is somewhat above zero, but the fluctuation between winter and summer is large enough. Like in Wisconsin.