Escaping set of an entire function

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Let f be an entire function of one complex variable. We denote by f^n the n-th iterate of f. Consider the set in the complex plane

$$I(f) = \{z : f^n(z) \to \infty, \quad \text{as} \quad n \to \infty\}.$$

It is known that this set is always not empty (unless f is a polynomial of degree 1), and the boundary of I is the Julia set. For polynomials f the set $I(f)$ is well understood: it is the basin of attraction of ∞.

For transcendental functions f, I conjecture that $I(f)$ cannot have bounded components. It is known that its closure cannot have bounded components, and the conjecture is known to be true for certain classes of entire functions. On the other hand, one cannot replace components by "path-connected components" as it is shown in [2].

References

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