**Math 511 Midterm Exam, Fall 2024**

NAME:

1. Let $A = (a_{i,j})$ be the inverse matrix to

$$\begin{pmatrix} 1 & 2 & 1 \\ 0 & 2 & 1 \\ 2 & 1 & 0 \end{pmatrix}.$$

What is $a_{3,2}$?

A. $-3$

B. 1

C. $-1$

D. $1/3$

E. $-1/3$

Ans.: A.

2. A system $A\mathbf{x} = \mathbf{b}$ with $m \times 2024$ matrix $A$ and $\mathbf{b} = (1, 0, \ldots, 0)^T$ has a unique solution. Which of the these statements *does not* follow from these conditions?

A. $N(A) = \{\mathbf{0}\}$

B. $\text{rank}(A) = 2024$

C. $m \geq 2024$

D. Columns of $A$ are linearly independent

E. $A$ is invertible

Ans.: E.

3. Let $A = LU$, where $L$ is lower triangular with 1's on the main diagonal, and $U = (u_{i,j})$ is upper triangular, and

$$A = \begin{pmatrix} 2 & 1 & 3 & 0 \\ -4 & 0 & -5 & 0 \\ 2 & 5 & 5 & 2 \end{pmatrix}.$$

What's $u_{2,4}$?

A. 2

B. $-1$

C. 1

D. 0

E. none of the above

Ans.: D.

4. Let $p(t) = t^2$. On the space of all polynomials of $t$ with real coefficients, which of these $T_k$ are linear operators (the circle means composition):

$$T_1(f) = pf, \quad T_2(f) = p \circ f, \quad T_3(f) = f \circ p, \quad T_4(f) = f + p.$$

A. All of them

B. None of them

C. $T_1$ and $T_3$

D. Only $T_2$

E. Only $T_1$

Ans.: C.

5. What is the projection of the vector $(1, 2)^T$ on the line $x = y$ in $\mathbf{R}^2$?

   A. $(3/2, 3/2)^T$

   B. $(2, 2)^T$

   C. $(2, 1)^T$

   D. $(\sqrt{2}, \sqrt{2})^T$

   E. None of the above

   Ans.: A

6. What is the angle between $f(t) = 1$ and $g(t) = t$ in the space of functions on the interval $0 \leq t \leq 1$ with the standard dot product?

A. $\arccos \frac{1}{2\sqrt{3}}$

B. $\arccos \frac{1}{\sqrt{3}}$

C. $\arccos \frac{1}{\sqrt{2}}$

D. $45^o$

E. None of the above

Ans.: E

7. Let $A$ be a $4 \times 4$ matrix in which the first two rows are not proportional (none of them is a multiple of another), the third row is the sum of the first two, and the fourth row is the difference of the first two. What are the dimensions of the four subspaces $R(A), C(A), N(A)$ and $N(A^T)$ (in this order)?

A. $3, 3, 0, 4$
B. $2, 2, 2, 2$
C. $3, 3, 0, 0$
D. $1, 1, 3, 2$
E. $1, 1, 2, 2$

Ans.: B

**True of false:**

8. The column space of a matrx has a unique basis, and this basis consists of certain columns of this matrix.

Ans.: F

9. Sums and products of symmetric matrices are always symmetric.

Ans.: F

10. In the vector space $\mathbf{R}^7$, there are two subspaces, both of dimension 4, whose only common vector is the zero vector.

Ans.: F