## Math 425/525 Midterm exam, Fall 2020.

NAME:

1. Evaluate

$$
2 i \log \frac{1-i}{1+i}
$$

Your answer should in the form $a+i b$ where $a$ and $b$ are real.
2. Describe and sketch the image of the sector

$$
\{z:|z|<1, \pi / 2<\operatorname{Arg} z<\pi\}
$$

under the Joukowski function $f(z)=(z+1 / z) / 2$.
3. Let $f$ be a non-constant analytic function in some region. Can $|f|^{2}$ be harmonic in this region? (The answer must be justified: either you give an example of such function, or explain why it does not exist).
4. Evaluate with any method

$$
\int_{\gamma} \frac{\sin z}{z^{2}-1}
$$

where $\gamma$ is the circle $\left\{2 e^{i t}: 0 \leq t \leq 2 \pi\right\}$.
5. Find a bounded harmonic function in the strip

$$
\{x+i y:-\infty<x<\infty, 0<y<1\}
$$

which takes the value 1 on the positive ray $[0,+\infty)$ and 0 on the rest of the boundary of the strip. Hint: use the exponential function to map the strip onto the upper half-plane
6. Which of the following statements are true:
a) If $u(z)$ is harmonic then $u(\bar{z})$ is harmonic.
b) If $f(z)$ is analytic then $f(\bar{z})$ is analytic.
c) If $f(z)$ is analytic then $\overline{f(\bar{z})}$ is analytic.
d) If $u$ is harmonic then $u^{2}$ is harmonic.
e) If $f$ is analytic then $f^{2}$ is analytic.

Here $\overline{x+i y}=x-i y$ is the complex conjugation.
No justification is necessary; each correct answer is worth 2 points.
7. Which of the following statements are true:
a) There is a branch of $\left(z^{2}-1\right)^{1 / 2}$ in the region $\{z:|z|>1\}$.
b) There is a branch of $\left(z^{3}-1\right)^{1 / 2}$ in the region $\{z:|z|>1\}$.
c) There is a branch of $\log \left(z^{2}-1\right)$ in the region $\{z:|z|>1\}$.
d) There is a branch of $\arccos z$ in the region $\{z:|z|<1 / 100\}$.
e) There is a branch of the inverse function to $J(z)=\left(z+z^{-1}\right) / 2$ in the plane with deleted segment $[-1,1]$.

No explanation necessary: every correct answer gives you 2 points.

