Math 425/525 Midterm exam, Fall 2020.

NAME:

1. Evaluate

$$2i\text{Log}\,\frac{1-i}{1+i}.$$

Your answer should in the form $a+ib$ where a and b are real.

2. Describe and sketch the image of the sector

$$\{z : |z| < 1,\ \pi/2 < \operatorname{Arg} z < \pi\}$$

under the Joukowski function $f(z) = (z + 1/z)/2$.

3. Let f be a non-constant analytic function in some region. Can $|f|^2$ be harmonic in this region? (The answer must be justified: either you give an example of such function, or explain why it does not exist).

4. Evaluate with any method

$$\int_\gamma \frac{\sin z}{z^2 - 1},$$

where γ is the circle $\{2e^{it} : 0 \leq t \leq 2\pi\}$.

5. Find a bounded harmonic function in the strip

$$\{x + iy : -\infty < x < \infty,\ 0 < y < 1\}$$

which takes the value 1 on the positive ray $[0, +\infty)$ and 0 on the rest of the boundary of the strip. *Hint: use the exponential function to map the strip onto the upper half-plane*

6. Which of the following statements are true:

a) If $u(z)$ is harmonic then $u(\overline{z})$ is harmonic.

b) If $f(z)$ is analytic then $f(\overline{z})$ is analytic.

c) If $f(z)$ is analytic then $\overline{f(\overline{z})}$ is analytic.

d) If u is harmonic then u^2 is harmonic.

e) If f is analytic then f^2 is analytic.

Here $\overline{x+iy} = x - iy$ is the complex conjugation.
No justification is necessary; each correct answer is worth 2 points.

7. Which of the following statements are true:

a) There is a branch of $(z^2-1)^{1/2}$ in the region $\{z : |z| > 1\}$.

b) There is a branch of $(z^3-1)^{1/2}$ in the region $\{z : |z| > 1\}$.

c) There is a branch of $\log(z^2-1)$ in the region $\{z : |z| > 1\}$.

d) There is a branch of $\arccos z$ in the region $\{z : |z| < 1/100\}$.

e) There is a branch of the inverse function to $J(z) = (z + z^{-1})/2$ in the plane with deleted segment $[-1, 1]$.

No explanation necessary: every correct answer gives you 2 points.