# Homework 9

**1.** a) For which positive integers $m$ is it possible that $2^m$ and $2^{m+1}$ have equal sums of digits in decimal system? *Hint*: Every number has the same residue modulo 3, as sum of its digits in decimal system. So, for example $721,020,013,214$ has residue 2 modulo 3.

   b) How to find out quickly, whether a given integer is divisible by 11?

**2.** a) Express a primitive root of 1 of degree 8 in the form $a + bi$ (your expression may contain radicals but no sines, cosines or exponentials).

   b*) Do the same for a primitive root of degree 5.

**3.** a) A complex number $w$ is a called primitive root of 1 of degree $d$, if all roots of 1 of degree $d$ are powers of $w$. Which roots of 1 of degree 12 are primitive, and which are not?

   b*) Is $\exp 7\pi i/60$ a root of 1 of degree 120? If yes, is it primitive?

**Definition**. Fourier matrix of size $N \times N$ is the matrix with elements $a_{i,j} = w^{-ij}$, where $w = \exp 2\pi\sqrt{-1}/N$ and $i, j = 0, \dots, N - 1$.

**4**. Write explicitly the Fourier matrix $6 \times 6$.

**5.** Let $A$ be the $4 \times 4$ Fourier matrix. Find $A^2$ and $A^4$

**6.** a) Show that the 4-th power of every Fourier matrix is a multiple of the unit matrix.

   b) How does the second power of a Fourier matrix look?

**7.** Find the Fourier transform of the vector $(2, 1, -2, 1)$.

**8.** According to Danielson and Lanczos, their first calculation at Purdue, took 10 min to find the FT of a 8-vector (all by hand, of course), 25 min for a 16-vector, 60 min for a 32-vector, and 140 min for a 64-vector. Plot these data, and discuss, whether are consistent with the theoretical result that the time is proportional to $N \log N$. Estimate the coefficient of proportionality. (Please do all calculations by hand:-)

**9.** Prove Parceval's identity:

$$\sum_{n=0}^{N-1} |F(n)|^2 = N \sum_{n=0}^{N-1} |f(n)|^2.$$

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**10.** Suppose that a vector $(a, b, c, d, e, f, g, h)$ has FT $(A, B, C, D, E, F, G, H)$. Find the vector (of dimension 4), whose FT is $(A, C, E, G)$.