MA 30300 Fall 2022 Section 211
Final Exam

NAME ______________________________ ID number ______________________________

VERSION 1

NO BOOKS, NOTES, CALCULATORS AND ELECTRONIC DEVICES

No partial credit for multiple choice problems

1. Solution $\mathbf{x}(t)$ of the initial value problem $\mathbf{x}' = \begin{bmatrix} 4 & -3 \\ 3 & 4 \end{bmatrix} \mathbf{x}; \quad \mathbf{x}(0) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ is

A. $e^{4t} \begin{bmatrix} 2\cos(3t) + 2\sin(3t) \\ \cos(3t) - \sin(3t) \end{bmatrix}$

B. $e^{4t} \begin{bmatrix} 2\cos(3t) - 2\sin(3t) \\ \cos(3t) - \sin(3t) \end{bmatrix}$

C. $e^{4t} \begin{bmatrix} 2\cos(3t) + \sin(3t) \\ \cos(3t) - 2\sin(3t) \end{bmatrix}$

D. $e^{4t} \begin{bmatrix} 2\cos(3t) + \sin(3t) \\ \cos(3t) + 2\sin(3t) \end{bmatrix}$

E. $e^{4t} \begin{bmatrix} 2\cos(3t) - \sin(3t) \\ \cos(3t) + 2\sin(3t) \end{bmatrix}$ correct

2. For the system

$$\frac{dx}{dt} = x - 2y + xy - 2$$
$$\frac{dy}{dt} = x - y + 1$$

the critical point $(2, 3)$ is

A. a stable node.

B. an unstable node.

C. a saddle point. correct

D. a stable spiral point.

E. an unstable spiral point.

3. For the mass-and-spring system

$$\frac{dx}{dt} = y$$
$$\frac{dy}{dt} = -5x - 2y + \frac{5}{4}x^3$$

the origin is

A. a stable spiral point. correct

B. an unstable spiral point.

C. a stable node.

D. an unstable node.

E. a saddle point.

4. Compute the inverse Laplace transform of $F(s) = \dfrac{e^{-s}}{s+2}$.
Here $u(t)$ is the unit step function.

A. $u(t-1)\, e^{-2t}$

B. $u(t-2)\, e^{-t}$

C. $u(t-1)\, e^{2}\, e^{-2t}$correct

D. $u(t-2)\, e^{-1}\, e^{-t}$

E. $u(t-1)\, e^{-2}\, e^{-2t}$

5. Solve the initial value problem $x'' + 4x = \sin(3t)$; $x(0) = x'(0) = 0$.

A. $x(t) = \dfrac{3}{10}\sin(2t) - \dfrac{2}{5}\sin(3t)$

B. $x(t) = \dfrac{3}{10}\sin(2t) - \dfrac{1}{5}\sin(3t)$ correct

C. $x(t) = \dfrac{3}{5}\sin(2t) - \dfrac{1}{10}\sin(3t)$

D. $x(t) = \dfrac{1}{5}\sin(2t) - \dfrac{3}{10}\sin(3t)$

E. $x(t) = \dfrac{1}{10}\sin(2t) - \dfrac{3}{5}\sin(3t)$

6. Find the convolution $t * e^t$. Hint: $\mathcal{L}\{t * e^t\} = \mathcal{L}\{t\}\,\mathcal{L}\{e^t\}$.

A. $e^t - t + 1$

B. $e^t + t + 1$

C. $e^t - t - 1$ correct

D. $e^t + t - 1$

E. $e^t - t\,e^t$

7. If $y' + 2y = u(t-2)e^{4-2t},\ y(0) = 1$, then $y(3) =$

A. $1 + 4e^{-2}$

B. $e^4 + e^2$

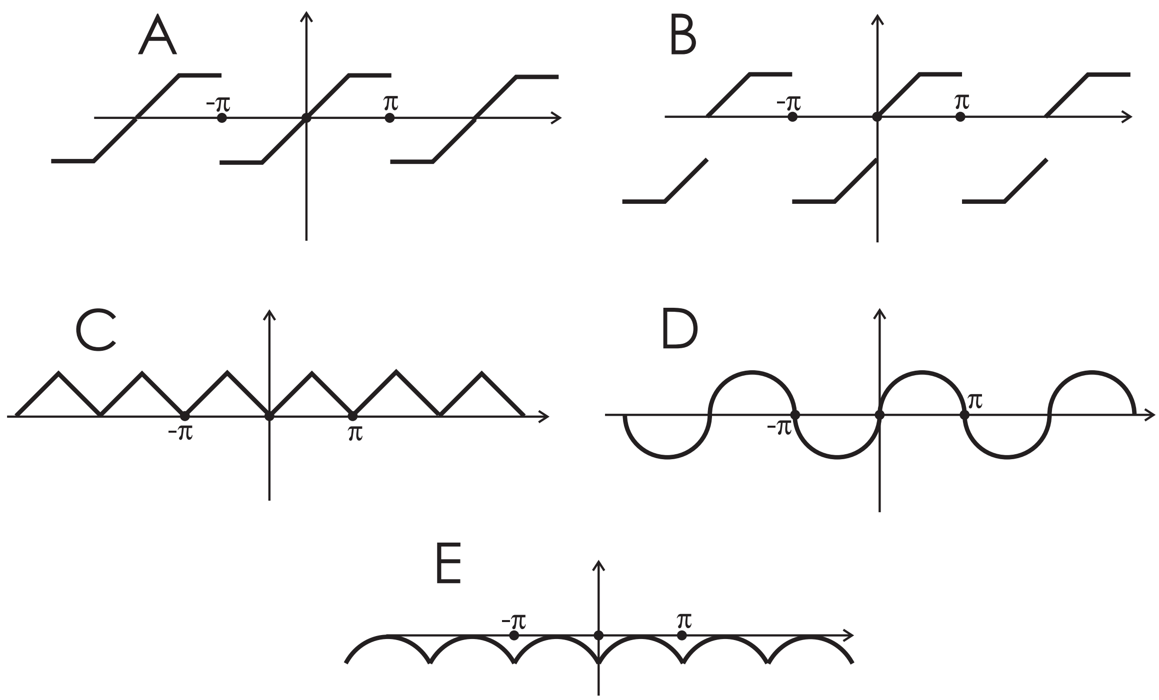
C. $e^{-8} - e^{-4}$

D. $e^{-6} + e^{-2}$ correct

E. $e^2 + e^{-2}$

8. Match the Fourier series $\dfrac{2-\pi}{\pi} + \dfrac{4}{\pi}\displaystyle\sum_{n=1}^{\infty} \dfrac{\cos 2nx}{1-4n^2}$ with one of the graphs given below.

Hint: use symmetry.



correct E

9. Given Fourier series for a 2π-periodic function $f(t)$, where $f(t) \equiv 0$ if $-\pi < t \leq 0$ and $f(t) = t$ if $0 < t < \pi$:

$$f(t) \approx \frac{\pi}{4} - \frac{2}{\pi} \sum_{n\,\mathrm{odd}} \frac{\cos(nt)}{n^2} + \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \sin(nt)}{n},$$

find the sum of the series $\displaystyle\sum_{n\,\mathrm{odd}} \frac{1}{n^2}$.

A. $\dfrac{\pi^2}{8}$correct

B. $\dfrac{\pi^2}{6}$

C. $\dfrac{\pi^2}{4}$

D. $\dfrac{\pi^2}{3}$

E. $\dfrac{\pi^2}{2}$

10. Given Fourier series for a 2π-periodic function $f(t)$, where $f(t) \equiv -1$ if $-\pi < t < 0$ and $f(t) \equiv 1$ if $0 < t < \pi$:

$$f(t) \approx \frac{4}{\pi} \sum_{n\,\mathrm{odd}} \frac{\sin(nt)}{n},$$

find the coefficient b_3 of the Fourier series for the steady periodic solution of the differential equation $x'' + x = f(t)$.

A. $-\dfrac{1}{3\pi}$

B. $\dfrac{1}{3\pi}$

C. $-\dfrac{1}{6\pi}$correct

D. $\dfrac{1}{6\pi}$

E. $\dfrac{2}{15\pi}$

11. If $u(x,t)$ satisfies the heat equation $\dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial x^2}$, $0 < x < \pi$, $t > 0$, with the boundary conditions $\dfrac{\partial u}{\partial x}(0,t) = \dfrac{\partial u}{\partial x}(\pi,t) = 0$ and initial condition $u(x,0) = 1 + \cos 2x$, then $u\left(\dfrac{\pi}{2}, \dfrac{3\ln 2}{4}\right) =$

A. 7/8correct

B. 15/16

C. 1

D. 17/16

E. 9/8

12. If $u(x,t)$ satisfies the wave equation $\dfrac{\partial^2 u}{\partial t^2} = 4\dfrac{\partial^2 u}{\partial x^2}$, $0 < x < \pi$, $t > 0$, with the boundary conditions $u(0,t) = u(\pi,t) = 0$ and initial conditions $u(x,0) = 2\sin 2x$, $\dfrac{\partial u}{\partial t}(x,0) = 0$, then $u\left(\dfrac{\pi}{2}, \dfrac{\pi}{4}\right) =$

A. -2

B. -1

C. 0 correct

D. 1

E. 2

13. Let $f(\theta)$ have the Fourier series $f(\theta) = 1 - \cos 2\theta + \sum_{n=1}^{\infty} \frac{1}{n^2} \sin n\theta$.

If $u(r, \theta)$ satisfies the 2-dimensional Laplace equation $\nabla^2 u(r, \theta) = 0$ in polar coordinates in the unit disk $\{r < 1\}$, with the boundary condition $u(1, \theta) = f(\theta)$, then $u\left(\frac{1}{2}, \pi\right) =$

A. 1/2

B. 3/4 correct

C. 1

D. 5/4

E. 3/2

14. The eigenvalue λ_2 of the Sturm-Liouville problem $y'' + \lambda y = 0,\ y(0) = 0,\ y'(\pi) = 0$, is

A. 1/4

B. 1

C. 3/2

D. 9/4 correct

E. 4