Math 511, Final exam, fall 2019

NAME:

Problems 1-4 are multiple choice: just circle the letters, no partial credit.

Problems 5-9 are partial credit, but please write your **answer**, if you obtain one, next to the problem.

1. Circle the letters corresponding to the statements which are true for all square matrices A, B of the same size.

A. If v is an eigenvector of A then v is also an eigenvector of e^A.

B. $e^{A+B} = e^A e^B$.

C. If A and B are similar then $\operatorname{tr} A = \operatorname{tr} B$.

D. If A is non-singular then it is diagonalizable.

E. If $A^{2019} = 0$ then A is singular.

2. Circle the letters which correspond to the statements which are true for all square 5×5 matrices A, B, C.

A. $\det(-A) = -\det(A)$.

B. $\det(3A) = 3\det(A)$.

C. $\det(A+B) = \det(A) + \det(B)$.

D. $\det(ABC) = \det(A)\det(B)\det(C)$.

E. Determinant of A does not change if the rows A are rearranged in the opposite order.

3. Circle the letters which correspond to true statements for all square matrices of the same size:

A. If A and B are Hermitian matrices then $A + B$ is Hermitian.

B. If A and B are Hermitian then AB is Hermitian.

C. If A and B are unitary then $A + B$ is unitary.

D. If A and B are unitary then AB is unitary.

E. If A is Hermitian and B is unitary then $B^{-1}AB$ is defined and is Hermitian.

4. Suppose that A is a real symmetric negative definite matrix of size 4×4, that is $x^T A x < 0$ for all $x \neq 0$. What conclusions can be made from this ? Circle the corresponding letters.

A. A is non-singular

B. All eigenvalues of A are strictly negative.

C. Determinant of A is negative.

D. All upper left minors of A are negative.

E. No row exchanges are required when bringing A to the upper triangular form by row operations.

5. For the matrix

$$\begin{pmatrix} 2 & 2 & -2 \\ 0 & 1 & 0 \\ 1 & 1 & -1 \end{pmatrix},$$

find the Jordan form and a Jordan basis.

6. Find e^{At} for the matrix

$$A = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}.$$

(Exponential is defined as a matrix solution $X(t)$ of the differential equation

$$\frac{d}{dt}X = AX, \quad \text{such that} \quad X(0) = I.)$$

7. Find the signature of the quadratic form

$$xy + yz + xz$$

8. Evaluate the determinant

$$\begin{vmatrix} x & y & y & y & y \\ y & x & y & y & y \\ y & y & x & y & y \\ y & y & y & x & y \\ y & y & y & y & y \end{vmatrix}.$$

9. The first and second columns of a rotation matrix A are

$$\begin{pmatrix} 1/3 \\ 2/3 \\ -2/3 \end{pmatrix}, \quad \text{and} \quad \begin{pmatrix} 2/3 \\ 1/3 \\ 2/3 \end{pmatrix}.$$

a) Find the third column of A.

b) How many solutions does this problem have?

c) If two columns a and b are given, what are the conditions on a and b for this problem to be solvable?