

Math 525, Final exam. Fall 2021

NAME:

1. Solve the equation

$$\tan z = 2i.$$

Write all solutions and make a picture of them.

2. Evaluate the integral

$$\int_0^{\infty} \frac{\cos z}{z^2 + 1} dz.$$

3. a) Find a conformal maps f of the region

$$\{z = x + iy : x > 0, y > 0, x^2 + y^2 < 1\}$$

onto the upper half-plane.

b) Is it possible to find such a conformal map with the additional properties $f(1) = 0$, $f(i) = 1$, $f(0) = \infty$? If your answer is positive, find it, if negative, explain why.

4. How many solutions does the equation

$$e^z = z^2 - 2$$

have in the left half-plane?

5. Evaluate the integral

$$\int_{|z|=4} \frac{e^z}{\sin z} dz.$$

6. Describe and sketch the set

$$\{z : |e^{z^2}| = 2\}.$$

7. a) Find the radius of convergence of the series

$$f(z) = \sum_1^{\infty} \frac{2^n}{n} z^n.$$

b) Find $f(z)$ explicitly (it is an elementary function).