MA 520 Midterm Exam Fall 2000

NAME_____ Student ID_____

1. (10 pts) Find a Fourier series for

$$f(x) = \begin{cases} \sin\theta, & \text{if } 0 \le \theta \le \pi \\ \theta, & \text{if } -\pi \le \theta < 0 \end{cases}$$

2. (5 pts) Find the sum of the Fourier series in Problem 1 at $\theta = 0, \pi/2, \pi$.

3. (10 pts) Find the best approximation in $L^2(0, \pi)$ to $f(x) = \cos x$ by a linear combination of $\sin x,\ \sin 2x,\ \sin 3x$.

4. (10 pts) Represent

$$\sum_{n=1}^{\infty} \frac{1}{(2n-1)^6}$$

as an integral of a polynomial over $[0, \pi]$. Do NOT evaluate the integral.

5. (10 pts) Use identity $(\theta^4)'' = 12\,\theta^2$ to find a Fourier series for $f(\theta) = \theta^4$.

6. (15 pts) Solve $u_t = u_{xx} + e^{-t} \sin x$ for $x \in [0, \pi]$ and $t \geq 0$ with the boundary conditions $u(0, t) = u(\pi, t) = 0$ and initial condition $u(x, 0) = \sin 2x$.
Hint: $(t\, e^{-t})' = e^{-t} - t\, e^{-t}$.