Let $z_n \to \infty$ be a sequence in te complex plane. Consider a meromorphic function

$$f(z) = \sum_{n=1}^{\infty} \frac{a_n}{z - z_n}, \quad \sum_{n=1}^{\infty} |a_n| < \infty, \quad \sum_{n=1}^{\infty} a_n \neq 0.$$

Is it true tat each such f must have zeros?

This is known under various additional conditions, for example

a) f is of finite order,

b) $a_n > 0$.