On combinatorial invariants of Minkowski spaces

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Abstract

The Steinitz's lemma (1913) asserts that given a finite set V in the unit ball of Minkowski (i.e. finite-dimensional real Banach) space E with sum 0, there is an ordering $V = v_1, ..., v_n$ such that $\|v_1 + ... + v_k\| \leq C$ for all $k = 1, 2, ..., n$. Here C depends on E, but not on V. Let $S(E)$ denote the minimal C in this assertion.

In 1963 A. Dvoretzky asked the question: "For a Minkowski space E and positive integer n, define $v(E, n) = \max\min \|\pm x_1 \pm x_2 \pm \ldots \pm x_n\|$, where the minimum is over all 2^n possible choices of + and − signs and the maximum is over all n-tuples of unit vectors. What can be said about the numbers $v(n, E)$?" Let $D(E) = \sup v(E, n)$ over all n.

In the talk, we study $S(E), D(E)$, their generalizations and specifications, some other isometric invariants of Minkowski spaces and relations between them.

Typical results:

1) For all $E, S(E) \leq \dim E$.

2) For all $E, D(E) \leq \dim E$.

3) For $1 \leq 2, D(l_p^d) = d^{1/p}$.