Singularities of implicit functions

A. Eremenko

April 4, 2015

Let F be an entire function of two variables, and suppose that for some (z_0, w_0) we have

$$F(z_0, w_0) = 0 \quad \frac{\partial F}{\partial w}(z_0, w_0) \neq 0.$$

Then by the implicit function theorem there is a holomorphic germ ϕ such that

$$F(z, \phi(z)) \equiv 0, \quad \phi(z_0) = w_0.$$

Stoïlov [1] proved that ϕ has the *Iversen property:* for every curve $\gamma : [0, 1] \to \mathbf{C}$ such that $\gamma(0) = z_0$ and every $\epsilon > 0$ there exists a curve $\gamma_1 : [0, 1] \to \mathbf{C}$ such that $|\gamma(t) - \gamma_1(t)| \leq \epsilon,\ 0 \leq t \leq 1$, and ϕ has an analytic continuation along γ_1. See also [2].

If F is of the form $F(z, w) = z - f(w)$ then the stronger *Gross property* is known: ϕ has an analytic continuation along almost every ray beginning from z_0.

Does the Gross property hold for every implicit function defined by an entire relation in two variables?

References

[1] S. Stoilov, Mathematica (Cluj) 12 (1936) 123–138.

[2] M. Tsuji, Potential theory in modern function theory, Maruzen, Tokyo, 1959.