Hints and Solutions, HW 2

1. Let $y(t)$ be the temperature of the carcass t hours after the boar was killed. According to the law of cooling

$$y' = k(40 - y),$$

where k is (unknown) coefficient of proportionality. This equation is separable and the general solution is

$$y(t) = 40 + Ce^{-kt}.$$

Using $y(0) = 100$, we obtain $C = 60$. Now the results of the ranger's measurements give

$$40 + 60e^{-kT} = 60 \quad \text{and} \quad 40 + 60e^{-k(T+1)} = 50,$$

where T is the time from the shot to the first measurement. Solving this system of equations we find that $k = \log 2$ and $T = \log 3/\log 2 \approx 1.5850$, that is one hour and thirty five minutes.

2. If at the time t (in hours) we have $y(t)$ kilograms of C, this means that $2y(t)/3$ kg of A and $y(t)/3$ kg of B have been consumed, thus $10 - 2y(t)/3$ kg of A and $20 - y(t)/3$ kg of B remains. Thus

$$y' = k(10 - 2y/3)(20 - y/3),$$

where k is a coefficient of proportionality, still unknown. The initial condition is $y(0) = 0$ (the water was pure, there was no C in the beginning).

The equilibria are $y_1 = 15$ and $y_2 = 60$.

The equation is separable, and its general solution is

$$\frac{60 - y}{15 - y} = Ce^{45kt}. \qquad (1)$$

Using the initial condition $y(0) = 0$, we obtain $C = 4$. To find k we use the condition $y(1/3) = 6$, which gives $e^{15k} = 3/2$. Using these values of C and k we solve (1) and find

$$y(t) = \frac{15(1 - (2/3)^{3t})}{1 - (1/4)(2/3)^{3t}}.$$

So, for $t = 1/2$ we have $y(1/2) \approx 7.9117$.

"Eventually" the output of the substance C will be 15 kg, all A will be consumed, and some B remain. Thus 99% of A will be consumed when 99% of C is made. To find when will this happen we write

$$.99 \times 15 = \frac{15(1 - (2/3)^{3t})}{1 - (1/4)(2/3)^{3t}},$$

which gives

$$t = \frac{1}{3\log(2/3)} \log \frac{.1}{1 - .99/4} \approx 3.5521,$$

that is approximately three hours and thirty three minutes.

3. a) and c) are homogeneous; b) is exact.

5. $b = 3$.

6. Just differentiate (assuming that y is a function of x):

$$e^y + xe^y y' - y^2 - 2xyy' + y' = 0,$$

or $y'(xe^y - 2xy + 1) + e^y - y^2 = 0$.