

Homework 12

1. Solve the Dirichlet problem for the unit disc with the boundary conditions 1 on the upper half of the unit circle and -1 on the lower half.
2. Solve the Dirichlet problem for a strip $\{z : 0 < |\Im z| < 1\}$ with the boundary conditions: 0 on the real line, 1 on the ray $\{x + i : x > 0\}$ and -1 on the ray $\{i + x : x < 0\}$.
3. Let

$$f(x) = \begin{cases} 0, & x < 0 \\ 1, & x \geq 0 \end{cases}$$

Using the Schwarz-Christoffel formula, find the conformal map of the region $\{x + iy : y > f(x)\}$ onto the upper half-plane with the boundary correspondence: $\infty \mapsto \infty$, $0 \mapsto 0$, $i \mapsto 1$. Describe precisely the branches of the functions that occur in your answer.

4. Find a conformal map of the upper half-plane onto the unit square $\{x + iy : |x| < 1, |y| < 1\}$. Leave your answer in the form of the integral. (This integral cannot be expressed in terms of elementary functions).
5. Let f be a conformal map of a triangle onto the upper half-plane. Describe all triangles for which the function f is meromorphic in the plane.
6. Consider two rectangles, one having two sides of lengths a, b , another having two sides of lengths a', b' . Suppose that there exists a conformal map of one of these rectangles onto another, sending vertices to vertices. Prove that $a/b = a'/b'$ or $a/b = b'/a'$.