1. An incident in a national park.
One morning a ranger in a national park found a carcass of a boar. An inspection showed that the boar was shot and died instantly when the bullet hit him. To make a case against a suspected poacher it was important to determine the time when the boar was killed. For this purpose the ranger made two measurements of the temperature of the carcass. The first measurement gave $60°$, and the second, one hour later, $50°$. The air temperature was constant that morning, and equal to $40°$. The body temperature of a boar (when he is alive) is known to be $100°$. How long before the first measurement was the boar killed?

2. Simplest chemical reactions. Two substances A and B react in water solution, and a substance C is produced as a result. Two units (of mass) of A and one unit of B produce three units of C. The speed of the reaction is proportional to the product of the present amounts of A and B. (Think, why this is so!).

Assume that 10 kg of A and 20 kg of B were initially dissolved in (pure) water, and 20 minutes later the water solution contained 6 kg of C.

a). Write the differential equation for the amount of C as a function of time. What initial condition is implied by the problem?

b). Find the equilibria and draw the phase portrait of this equation.

c). Find the solution of the equation, satisfying the initial condition.

d). How much of C will be present in 30 minutes after the beginning of the process?

e). How much time (from the beginning of the process) is needed for 99% of A to react?

3. Find general solutions of the following differential equations. You may leave your answers in implicit form, or even with integrals, if you cannot evaluate them.

a) $y' = (x^2 - y^2)/(x^2 + y^2)$,

b) $y'(\sin x + x^2 e^y + 2) + y \cos x + 2xe^y = 0$,

c) $(y^2 + 2xy)dx - x^2 dy = 0$.

4. Verify that $(xy^2)^{-1}$ is an integrating factor for $(y^2 + xy)dx - x^2 dy = 0$, and solve this equation.

5. Find all values of b, for which the differential equation

$$(xy^2 + bx^2 y)dx + (x + y)x^2 dy = 0$$

is exact, and solve it with this value of b.

6. Find a differential equation, whose general (implicit) solution is
$xe^y - xy^2 + y = C$.