Homework 4

1. Parachute. There are two forces acting on a parachute with a parachutist: the force of gravity and the air resistance. Suppose that the air resistance is kv^2, where v is the speed of descent, and k is a positive constant.

a) In which units this k is to be measured?

b) Write a differential equation describing the parachute motion, assuming that there is no wind.

c) Find an expression for $v(t)$.

d) We know from experience that a parachutist reaches some limit speed soon after s/he jumps. (The reason why parachutes are used at all is to reduce this limit speed). Explain why is this so by looking at your expression for $v(t)$.

e) Express this limit speed in terms of the mass of the parachutist (it includes the parachute and all other equipment), acceleration of gravity and the constant k. If one parachutist is twice as heavy as another, and they use similar parachutes, so that their k's are the same, then what will be the ratio of their limit speeds?

2.

a) Consider the differential equation $y'' + 4y = 0$. Using a method of your choice, find its general solution. Convert this equation into a first-order system. Sketch the corresponding vector field, and the phase portrait of this system.

b) Repeat all these steps for the equation $y'' + 4y + 4 = 0$. Describe the phase portraits of both systems in words.

3. Also do problems p.151: 7, 8, 23; p. 181: 14, 17, 18.