Homework 5

1. A projectile of mass m is fired vertically from a cannon with initial speed v_0. The air resistance has magnitude kv^2, where k is a constant. It is clear that the projectile will accent for some time, until it reaches some maximal altitude, and then will fall back. Express the time of accent in terms of m, v_0, k and the acceleration of gravity g. If v_0 is increased to a very large value, what happens to this time of accent? Can it be made arbitrarily large?

2. Convert the differential equation of the pendulum $x'' + \sin x = 0$ into a first order system. Write the differential equation for the trajectories, and solve it. Interpret your solution as an energy conservation law. Try to sketch the phase portrait.

3. Find general solutions of the following differential equations:

$$y'' + 3y' - 10y = 0, \quad y'' + 3y' + 10y = 0, \quad y^{IV} - y = 0.$$

5. Consider the case of Lanchester's model, when a guerilla force fights regular troops. Assume there are no losses from other reasons than combat, and no reinforcements. Tell which side will win from their initial numbers and combat efficiencies.