Little Jo and her pig. Solution

Following the hints, we denote by $y = f(x)$ the function whose graph is Jo's trajectory. The time of Jo's arrival to a point (x, y) on this graph equals to the arclength of this graph from $(-1, 0)$ to (x, y) divided by her speed. Denoting her speed by k, and using the familiar formula from Calculus for the arclength, we obtain the expression

$$t = \frac{1}{k}\int_{-1}^{x}\sqrt{1+(y')^2}dx.$$

(We prefer not to use any numerical data like $k = 1.5$ till the very end).

Now if Jo is at the point (x, y) at time t then the pig is at the point $(0, t)$, so Jo's direction is given by the vector $(-x, t - y)$, whose slope is $(y - t)/x$, and this should be equal to $y' = dy/dx$. Thus we obtain

$$y' = \left(y - \frac{1}{k}\int_{-1}^{x}\sqrt{1+(y')^2}dx\right)/x.$$

To transform this into a differential equation, we multiply by x, differentiate with respect to x and simplify. The result is

$$kxy'' = -\sqrt{1+(y')^2}.$$

Setting $v = y'$, we obtain a first-order separable equation

$$\frac{dv}{\sqrt{1+v^2}} = -\frac{1}{kx}.$$

The integral in the RHS (the so-called 'Long Logarithm') is somewhat clumsy, so I prefer to use hyperbolic functions instead, do decrease the chance of an error in calculations.

$$\sinh^{-1} v = -\frac{1}{k}\log|x| + C.$$

When the race starts, we have $x = -1$ and $v = y' = 0$, because Jo's direction at the starting point is along the x-axis. Plugging this we obtain $C = 0$. Thus

$$v = \sinh(-\frac{1}{k}\log|x|) = \frac{1}{2}\left(|x|^{-1/k} - |x|^{1/k}\right),$$

by definition of the hyperbolic sine. Now

$$y' = \frac{1}{2}\left(|x|^{-1/k} - |x|^{1/k}\right),$$

and it remains to evaluate the integral. In fact we are interested only in $y(0)$, where Jo reaches the vertical axis. The result is

$$\begin{aligned} y(0) &= \frac{1}{2}\int_{-1}^{0}(-u)^{-1/k} - (-u)^{1/k}du \\ &= \frac{1}{2}\left(\frac{k}{k-1} - \frac{k}{k+1}\right) \\ &= \frac{k}{k^2-1}. \end{aligned}$$

Jo will catch the pig if $y(0) \leq 1$. This means $k^2 - k - 1 \geq 0$, or $k \geq (1+\sqrt{5})/2 \approx 1.618$. Thus if $k = 1.5$, the pig will escape. Notice that running straight to the hole requires the speed of only $\sqrt{2} \approx 1.414$ to be at the hole in time to intercept the pig.