

## Laplace Transform

$f(t), t \geq 0$	$F(s) = L(f)(s)$
$e^{at}$	$1/(s-a), s > a$
$t^n$	$n!/s^{n+1}, s > 0$
$\sin bt$	$b/(s^2 + b^2), s > 0$
$\cos bt$	$s/(s^2 + b^2), s > 0$
$u_a(t), a > 0$	$e^{-as}/s, s > 0$
$\delta_a(t), a > 0$	$e^{-as}, s > 0$
$f_1(t) + f_2(t)$	$F_1(s) + F_2(s)$
$cf(t)$	$cF(s)$
$f(at)$	$a^{-1}F(s/a)$
$f'(t)$	$sF(s) - f(0)$
$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
$f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \dots - f^{(n-1)}(0)$
$e^{at}f(t)$	$F(s-a)$
$t^n f(t)$	$(-1)^n(d^n/ds^n)F(s)$
$f_1 \star f_2$ (convolution)	$F_1F_2$ (usual product)

## Partial fractions

1. Let  $p(s)/q(s)$  be a rational function. If  $\deg p < \deg q$  go to the next step. If  $\deg p \geq \deg q$ , divide with a remainder to obtain  $p(s)/q(s) = h(s) + r(s)/q(s)$ , where  $h$  is a polynomial and  $\deg r < \deg q$ . Then do the next step on  $r(s)/q(s)$ .

2. Factor the denominator  $q(s)$ . If you are working over complex numbers, all factors should be of degree 1, if over real numbers, all factors have to be of degree 1 or 2. Go to step 3 or 3'.

3. To each factor  $(s-a)^n$  in the denominator correspond the following partial fractions

$$\frac{A_1}{s-a} + \frac{A_2}{(s-a)^2} + \dots + \frac{A_n}{(s-a)^n}.$$

If you are working over complex numbers, this is enough. Write the partial fractions with undetermined coefficients  $A_j$  and then find these coefficients by solving a system of linear equations.

3'. Over the real numbers, to each quadratic factor to the form  $(s^2 + bs + c)^n$  corresponds a sum of partial fractions of the form

$$\frac{A_1s + B_1}{s^2 + bs + c} + \frac{A_2s + B_2}{(s^2 + bs + c)^2} + \dots + \frac{A_ns + B_n}{(s^2 + bs + c)^n}.$$