

Remarks on *Nevanlinna functions with real zeros*:

This is a very good paper and I enthusiastically recommend its publication. I do have a number of corrections and stylistic comments.

p.1 l'.9:

"We are interested in solutions all of whose roots are real."

p.2 l.10:

Perhaps you should mention that in the Titchmarsh result there are some choices of  $c$  which admit solutions with infinitely many zeros *all* of which are real.

p.3 l.3:

Obviously the word "them" refers to the word "roots" but the grammatical construction is ambiguous. How about "For odd  $d$  there are infinitely many roots of  $w$  all of which lie on a ray."

p.5 l.3ff:

The reader should be given some references to a), b) and c), say Nevanlinna's *Analytic Functions*. This is not easy stuff if one is looking at it for the first time. Part c) is particularly confusing. Why for instance is  $\tilde{X}$  a sphere in c)?

p.5 l'.9:

Give a reference which explains why the number of logarithmic singularities is at least two or explain it yourself. A reader unfamiliar with this will already be pretty frustrated.

p.7 l.11:

Explain what you mean by dimension.

p.7 l'.6:

You showed on page 5 that  $k$  is either one or infinity and since you are dealing with  $N$ -surfaces this is all you will ever need. All of a sudden  $\pi$  is not a local homeomorphism and  $k$  might be an integer greater than 1. This adds an unnecessary level of complexity to the discussion on Speiser graphs. You are dealing with very special graphs and should only discuss them.

p.7 l'.1:

Define topologically holomorphic.

p.8 l.10:

"Such a map"

p.8 l.11:

I would skip the part about "ramified over base points" and just deal with  $N$ -surfaces here.

p.8 l.15:

Define isotopy class  $\Gamma$  with fixed set  $A$  or leave the whole paragraph out.

p.8 l'.1:

This is the first time you talk about multiple edges. Explain more clearly where they come up and why you can essentially ignore them.

p.9 l.10:

A tree  $T(S)$  that represents an  $N$ -surface is the union of its logarithmic ends. There is no need to talk about subtrees.

p.10 l'.14:

"Let  $a$  be a real base point." You have defined the  $a$ 's as *base* points (section 2), but there are many times throughout the paper where you refer to them as *basis* points. You should find all these instances and make the correction. I won't point these instances again.

p.10 l'.12:

Replace "identical" with "the identity."

p.11 l.17:

A. and B. confuse me. The graph in Figure 2 has exactly one edge on the axis, the rest of the logarithmic end is away from the axis. This seemingly contradicts A. Your figures also have edges that intersect the real axis at only a vertex, seemingly contradicting B. I must be missing something.

p.11 l'10.

I don't think you mean that the third assumption excludes the cases when the number of real zeros is finite. Indeed you have figures that represent functions with finitely many real zeros and yet the graph doesn't have  $0^+$  and  $0^-$  in it. You might say something like, "The third assumption causes no real loss of generality. In a graph where we are forced to use the labels  $0^+$  and  $0^-$ , there must be two non-zero real logarithmic singularities. Then  $f$  has non-zero limits along the real axis.....and thus  $f$  has only finitely many real zeros."

p.12 l.2

The interchanging of  $f$  and  $\pi$  is confusing.

p.12 l.8

Explain very clearly why a vertex not on a face labeled by 0 would

force a non real zero.

p.13 l'.8

Is the term "almost symmetric" standard? If not, a term like "anti-symmetric" might say it better.

p.13 l'5

Although I don't disagree with your statement "It is easy to see...", it is still worth giving the reader a little better explanation so that he or she pays attention to it and doesn't just blow it off as a remark made at the end of a paper. It is too nice a result for that.

p.14-18

In the captions for the figures it might be nice to tell the reader specifically what  $d$  is, e. g. on page 14 write  $d = 12 \equiv 0(mod4)$ . Also the captions on page 14 and 15 are more descriptive than the others. For example on page 14 you have "the sequence of zeros....directions." Why not do that on pages 16-18 as well?