Let f be a topologically holomorphic map from the plane to the sphere. This means that at every z_0 in the plane, f is either a local homeomorphism, or behaves like $z \mapsto (z - z_0)^n$ for some positive integer n. The points with $n \geq 2$ are called critical points, and the values of f at those points are called critical values. A point a in the sphere is called an asymptotic value if there is a curve $\gamma(t)$, $0 \leq t < 1$ such that $\gamma(t) \to \infty$ and $f(\gamma(t) \to a$ as $t \to 1$.

The standard conformal structure on the sphere pulls back via f to the plane, and the plane with this conformal structure becomes a Riemann surface. BY the Uniformization theorem, this Riemann surface is equivalent either to the complex plane (parabolic type) or to the unit disk in the complex plane (hyperbolic type).

The type depends on f. Consider the case when there are no asymptotic values and all critical values belong to some Jordan curve Γ in the sphere. Such a curve is called a base curve. The preimage $f^{-1}(\Gamma)$ in the plane is called a net. Two nets are called equivalent if there is a homeomorphism of the plane which takes one into another. The general question is

when the equivalence class of the net determines the conformal type?

A surprising result of Geyer and Merenkov [1], answering a question of E. B. Vinberg is that the square grid net does not determine the type. Some functions with the square grid net are elliptic (parabolic type), but there are others whose natural domain is the unit disk.

An interesting additional condition which can influence the answer is the symmetry: we say that f is real, if $f(\bar{z}) = \overline{f(z)}$. In this case, the net is symmetric with respect to $z \mapsto \bar{z}$.

Does the square grid net determine the parabolic type in the presence of the symmetry condition? (We assume that the real axis belongs to the net).

Now consider the "target net" which consists of the real line and infinitely many circles $|z| = r_k,\ r_k \to \infty$.

Does the target net in the presence of symmetry determine parabolic type?

References

[1] L. Geyer and S. Merenkov, A hyperbolic surface with a square grid net, J. Anal. Math. 96 (2005), 357–367.