Old exam Problems

1. Solve the following equations and initial value problems:

$$(1 + y^2)^2 dt = y \cos^2 t \, dy,$$

$$(3x^2 e^{3y} + y \sin(xy) + 3x^2)dx + (3x^3 e^{3y} + x \sin(xy) + 2y)dy = 0,$$

$$ty' + (1 + t)y = t^3 e^{-t},$$

$$\frac{dy}{dx} = \frac{x^4 + y^4}{xy^3},$$

$$(y - t^2 y)\frac{dy}{dt} = t, \quad y(2) = -4,$$

$$t^2 \frac{dy}{dt} = 2ty + 3, \quad y(1) = 5.$$

$$\frac{1}{t}\frac{dy}{dt} = 2t^{-2}y + t^3,$$

$$2\frac{dy}{dt} = y^2 - 5 + y^2 t^2 - 5t^2, \quad y(0) = -1.$$

2. An object of mass 1/32 slugs is thrown vertically upward from the surface of the earth with an initial speed of 2 feet per second. Air resistance is equal to $2|v|$, where v is the speed. Assuming the acceleration of gravity is 32 feet per second, find:
a) The differential equation for the speed as a function of time, and the initial condition implied by the problem.
b) The solution of this initial condition problem.
c) The time of accent.

3. A tank initially contains 5 lb of salt disolves in 100 gal of water. Water containing 2 lbs/gal of salt solution flows into the tank at the rate of 3 gal/min and the well stired solution flows out at the same rate. Find the formula for the amount of salt in the tank at any given time.