

Lectures on geometric theory of meromorphic functions

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Lecture 1. Introduction, Singularities, Results of Seibert, Speiser class and line complexes, Cellular functions and nets, Theorems of Gross and Iversen, Sullivan question, Merenkov result about square nets (mention), My partial result about target net and conjecture.

Prove: Gross' theorem.

Sources: Mypapers/Lectures/lects.tex, p. 1-8, Merenkov's paper, seibert-conspect.tex

Lecture 1a. Hyperbolic type criteria. Theorems of Picard, Bloch, Valiron, Ahlfors Lemma, Robinson, Zalcman Lemma, 5 islands, Bonk-Eremenko, Normal and Bloch functions.

Prove: Ahlfors Lemma, Robinson, Zalcman, 5 islands, Bloch.

Sources: lect.pdf p. 9-12, Geometric theory (mich.pdf) p. 2-5, extended version, Robinson, My paper with Bonk.

Lecture 2. Parabolic type criteria. Ahlfors, Kobayashi, Wittich, Milnor, Nevanlinna conjecture, Schramm-Merenkov.

Prove: Ahlfors criterion. Benjamini-Schramm-Merenkov construction.

Sources: Mypapers/Lectures/lects.tex, p. 12-19. Schramm-Merenkov.

Lecture 2a. Introduction to Nevanlinna theory. Direct and indirect singularities. Three theorems about them: DCA, Heins, BE. Applications to derivatives, Set of as values.

Prove: First Main Theorem, DCA (Carleman inequality) Heins.

Sources: Mypapers/Lectures/weizmann.tex (18 pages!)
Mypapers/Lectures/sing1.tex (16 pages), Mypapers/Lectures/lects.tex (17-21), BE w/out complete proof.

Lecture 3. Class S, Applications to dynamics, Struct stability, Applications to ODE, zero distribution of eigenfunctions (w Shapiro), Extremal problems: critical values, Length of lemniscate, Markov inequality. 6smith.

Prove: Lyubich lemma. Nevanlinna theorem?

Sources: E-Lyubich, E-Gabrielov, stokes.tex, 6-smith, E-shapiro.

Lecture 4. MacLane class. Nets. Applications, Shapiro-Shapiro. Some extremal problems: Hayman's question, Semmler.

Prove: MacLane-Vinberg theorem.

Sources: Mypapers/Lectures/vinberg.tex (4p), EG, stokes.tex, Mypapers/Lectures/mich.tex, Erem-Yuditskys (combs).

Lecture 4a Bank-Laine functions.

Sources: BE1, BE2, Bergw-Eremenko-Rempe, Mypapers/Lectures/talk3a.tex, talk3.pdf, 11p.