In problems 1-3 you do not have to show work. In problems 4-6 you should show work.

1. (15) Circle the letters corresponding to statements which are true for any two square matrices A and B such that $AB = 0$:

A. Either $A = 0$ or $B = 0$.

B. $\mathcal{C}(A) \subset \mathcal{N}(B)$.

C. $\mathcal{C}(A^T) \supset \mathcal{N}(B^T)$.

D. $BA = 0$.

E. Either $A = 0$ or B is singular.

2. (15) Which of the following is a subspace of the vector space $C[0, 1]$ of continuous functions on $[0, 1]$? Circle the letters corresponding to correct answers.

A. Functions with $f(1) = 0$.

B. Functions with $f(0) = 1$.

C. Functions with $\int_0^1 f(x)\,dx = 0$.

D. Functions with $f(0) = f(1)$.

E. Functions with $f(0) \le f(1)$.

3. (15) Which of the following is a linear transformation T of the space of 3×3 matrices? Circle the letters corresponding to correct answers.

A. $T(A) = U$ where U is the row echelon form of A.

B. $T(A) = A^{-1}$.

C. $T(A) = A^T$.

D. $T(A) = P$ where $p = Pb$ is the orthogonal projection of b to $\mathcal{C}(A)$.

E. $T(A) = E_{2,1}(3)A$ where $E_{2,1}(3)$ is an elementary matrix.

4. (20) Let

$$A = \begin{pmatrix} 1 & 2 & 2 \\ -6 & -11 & -12 \\ 1 & -4 & 3 \end{pmatrix}.$$

a) Factor $A = LU$.

b) Find A^{-1}.

5. (15) Let T be the transformation of the space $\mathbf{P}_2$ of polynomials of degree 2 defined as $T(p(x)) = p(x) - p(1)$.

a) Write the matrix of T in the basis 1, x, x^2 of $\mathbf{P}_2$.

b) Find the rank of T.

c) Find the fundamental subspaces $\mathcal{C}(T)$ and $\mathcal{N}(T)$.

6. (20) Find the projection of $b = \begin{pmatrix} 10 \\ 20 \\ 30 \end{pmatrix}$ onto the column space of $A = \begin{pmatrix} 1 & 1 \\ 1 & -1 \\ -2 & 2 \end{pmatrix}$.

7. (30) Find the distance from the origin to the plane $x + y + 2z = 3$.

8. Is it true that for every square matrix we have

$$r(A) = r(A^T A) = r(AA^T)$$

where $r(.)$ is the rank?
If true, explain why; if not, give a counterexample.
Hint: Consider $N(A)$ and $N(A^T A)$.