Let F be a polynomial in two variables (with complex coefficients). Prove that $F(z, \overline{z})$ is analytic if and only if F does not depend on the second variable.

For examples, polynomials $z^n \overline{z}^m$ are not analytic unless $m = 0$.

Prove that every polynomial of x and y can be written as a polynomial of z and $\overline{z}$. Here $z = x + iy$, of course.