Practice problems set 1

February 9, 2021

1. Inspect table 1 on p. 26–28 and tell which of the Fourier series 1–20 converge *uniformly* on the interval where the function in the left column is defined.

2. Let $f(x) = e^x$, $0 \le x \le \pi$, and let $F(x)$ be the sum of its sine Fourier series. Find $F(0)$.

3. Which of these functions belongs to $L^2(-\infty, +\infty)$:
   a) $(\sin x)/x$,
   b) $(\sin \sqrt{|x|})/\sqrt{|x|}$,
   c) $x^2 e^{-|x|}$,
   d) $x/(1+x^2)$.

3. Consider the following sequences of functions $f_n(x) = x^n$ and $g_n(x) = \sqrt{n}x^n$, defined on the interval $(0, 1)$. Which of these two sequences converge to 0
   a) at every point of the interval $(0, 1)$
   b) uniformly on the interval $(0, 1)$.
   c) in the sense of $L^2(0, 1)$.

4. Consider the sequence of functions $f_n(x) = e^{-|x-n|}$ on the whole real line. As $n \to +\infty$, does this sequence converge:
   a) at every real pont?
   b) uniformly on the whole real line?
   b) in $L^2(-\infty, +\infty)$?

5. Which of the following sequences are sine Fourier coefficients of some function $f \in L^2(0,1)$?

a) $b_n = 1/n$
b) $b_n = 1/\sqrt{n}$
c) $b_n = 1/(\sqrt{n}\log(n+1))$.

6. Using Table 1 on p. 26-28, find the sum of the series

$$\sum_{n=0}^{\infty} \frac{1}{(2n+1)^2}.$$