MATH 520, Spring 2007, Comments on the quizz

1. Write the general solutions of the following differential equations:

  a) $y' = -2y$, $\quad y(t) = Ce^{-2t}$.

  b) $y'' + 9y = 0$, $\quad y(t) = C_1 \cos(3t) + C_2 \sin(3t)$.

  c) $y'' - 9y = 0$, $\quad y(t) = C_1 e^{3t} + C_2 e^{-3t}$.

  Here C, C_1 and C_2 are arbitrary constants.

  Comment. *Firm* knowlede of this is a crucial prerequisite to 520. I mean, if you did not answer this correctly, you should consider switching to another course before it is too late. I mean these things will not be taught in 520 but they are absolutely required.

2. For the equations a) and b), name at least one physical system modeled by this equation.

  a) radioactive decay, cooling of an owen;
  b) load on a spring, pendulum, electric oscillator, any small oscillatory motion of a system with one degree of freedom.

3. Find a and b if $a + bi = \exp(\pi i/2)$, where $i = \sqrt{-1}$.

  $a = 0$, $b = 1$.
  Comments. Complex numbers and functions will play a very important role in this course. So this (and the next) problem are related to an important prerequisite. I recommend Appendix 2 to our textbook, to refresh your memory.

4. What is the sum of the infinite series

$$1 - x^2/2 + x^4/24 - x^6/720 + \ldots + (-1)^n x^{2n}/(2n)! + \ldots ?$$

This is cosine. See comments to Problem 3.

5. Is the number $10^{2007} + 1$ divisible by 3? If not, what is the remainder?

A number, when divided by 3, gives the same remainder as the sum of its digits. *Prove* this yourself, this is very easy. Same holds for division by 9. Prove it too. In particular, a number is divisible by 3 (by 9) if and only if the sum of its digits is divisible by 3 (or by 9). Try to find a similar criterion of divisibility by 11.

6. How many pixels are there in your computer screen, approximately ?

No comments.

7. What's the radius of the Earth, approximately, in your favorite units ?

It came as a big surprise to me how few students could answer this, even to ONE significant digit, many had even the ORDER of magnitude wrong. Of course, strictly speaking, the knowledge of such things, or the knowledge that the Earth is round, for example is formally *not required* in this course.