

In problems 1-8, for the given function determine the type of singularity at the given point. The answer should be of the form a,b,c,d, where

a: removable

b: pole

c: essential

d: not an isolated singularity

1.

$$\frac{1 - \cos z}{z^2}, \quad z = 0.$$

Answer: a

2.

$$\frac{\operatorname{Log} z}{z - 1}, \quad z = 1.$$

Answer: a ($\operatorname{Log} 1 = 0$).

3.

$$\operatorname{Log} z, \quad z = 0.$$

Answer: d

4.

$$\sin\left(\frac{\pi}{z^2 + 1}\right), \quad z = i.$$

Answer: c

5.

$$\cot z - \frac{1}{z}, \quad z = 0.$$

Answer: a

6.

$$z^2 \cot(1/z), \quad z = 0.$$

Answer: d (there are infinitely many poles at $1/\pi n$ accumulating to 0).

7.

$$\frac{z}{e^z + 1}, \quad z = \pi i.$$

Answer: b

8.

$$\frac{\cos z - 1 - \sin z^2}{z^4}.$$

Answer: b

In problems 9-10, find the residue, and enter it after the = sign

9.

$$\operatorname{res}_0 z^{-4} \cot z = -1/45.$$

10.

$$\operatorname{res}_0 (z^2 - 1) \exp(1/z) = 5/6.$$