

Rational curves with real inflection points

Let $f = (f_0, \dots, f_n)$ be a vector of complex polynomials. Suppose that the Wronskian determinant $W = W(f_0, \dots, f_n)$ has only real zeros. Then there exists a matrix $A \in GL(n+1, \mathbf{C})$, such that the vector Af consists of real polynomials.

This was conjectured by B. and M. Shapiro. The conclusion is not true if we only assume that W is a real polynomial. The conjecture is known to be true for $n = 1$ [1].

[1] A. Eremenko and A. Gabrielov, Rational functions with real critical points and B. and M. Shapiro conjecture in real enumerative geometry, *Ann. Math.*, 155 1 (2002).