

Problem 30.

Prove the general form of the Ahlfors–Schwarz Lemma.

I recall that a density λ is called ultrahyperbolic if λ is continuous and $\lambda(z) \geq 0$, and for every z_0 there exists a neighborhood V of z_0 and a smooth function λ_0 in V , satisfying $\Delta \log \lambda_0 \geq \lambda_0^2$, and such that $\lambda_0(z_0) = \lambda(z_0)$ and $\lambda_0(z) \leq \lambda(z)$ in V .

Then you have to prove, following the idea of the proof of the special case given in class, that every ultrahyperbolic density in the unit disc satisfies $\lambda \leq \rho$ where $\rho(z) = 2(1 - |z|^2)^{-1}$.